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FRONTISPIECE.

THE CIRCLE, SQUARE, AND TRIANGLE;
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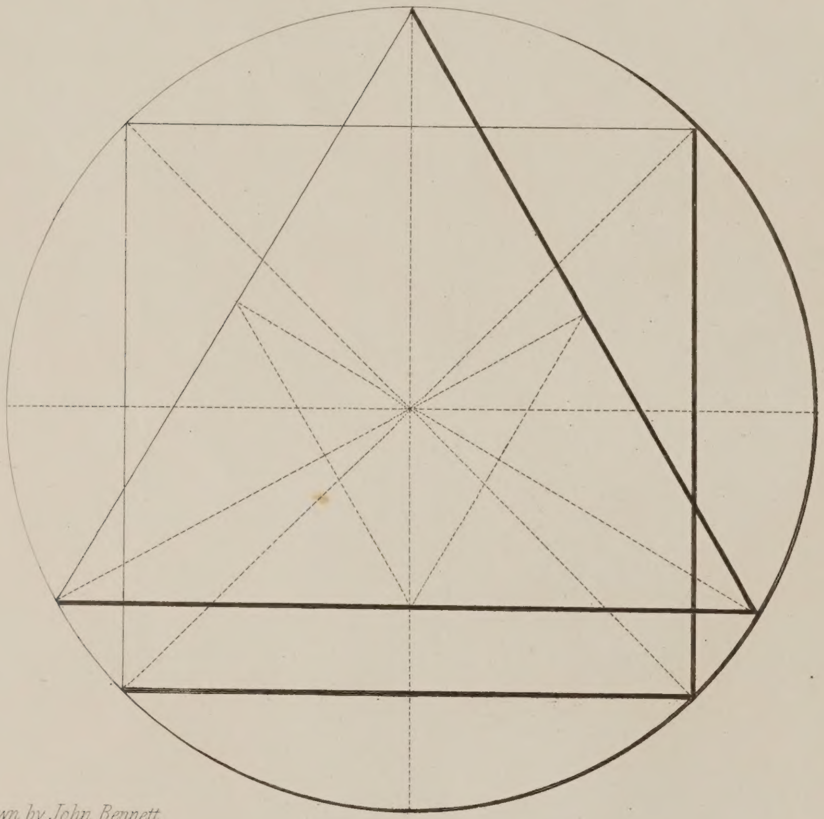


Drawn by John Bennett.

London: John Bennett, 4, Three Tun Passage, Ivy Lane, Paternoster Row.

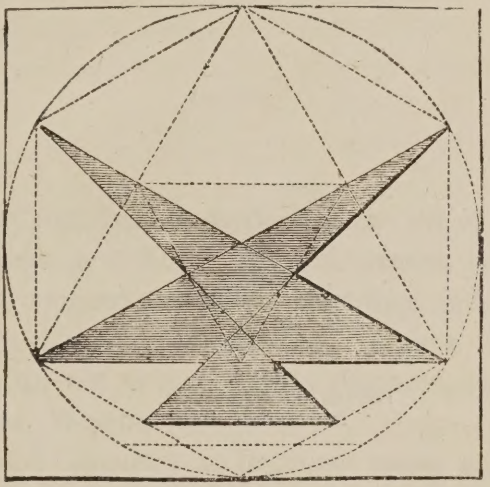
ORIGINAL

GEOMETRICAL ILLUSTRATIONS;

OR

THE BOOK OF LINES, SQUARES, CIRCLES, TRIANGLES, POLYGONS, &c.

SHOWING



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Containing also a variety of Useful Information,

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A COMPLETE INSTRUCTOR

TO THE

MOST USEFUL SCIENCE OF GEOMETRY AND MENSURATION.

BY JOHN BENNETT, ENGINEER,

AUTHOR OF THE ARTIFICER'S COMPLETE LEXICON, LABOUR PRICES FOR BUILDERS, &c.

LONDON: JOHN BENNETT, 4, THREE-TUN PASSAGE, IVY LANE, PATERNOSTER ROW;
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PREFACE.

GEOMETRY originally signified no more than the art of measuring the earth, or any distance or dimension within it ; but, at present, it denotes the science of magnitude generally, comprehending the doctrines and relations of whatever is susceptible of augmentation, or diminution. Hence to Geometry may be referred the consideration not only of Lines, Surfaces, and Solids, but also of Time, Velocity, Number, Weight, &c.

Plato thought the word Geometry an improper name, and accordingly substituted in its place the more extensive one of Mensuration ; and after him others gave it the title of Pantometry, as demonstrating not only the quantities of all manner of magnitudes, but also their qualities, ratios, positions, transformations, relations, &c. And Proclus describes it as the knowledge of magnitudes, figures, &c., and their limitations, also of their motions and affections, of whatsoever kind or description.

This science had its origin amongst the Egyptians, who were in a manner compelled to invent it to remedy the confusion which generally happened in their lands from the inundations of the river Nile, which carried away all the boundaries and effaced all the limits of their possessions. This invention at first consisted in measuring land, that every person might be in possession of the just quantity which belonged to him. It was then termed Geometry, or the art of measuring land : it is therefore probable that the draughts and schemes, which they were compelled to make, caused them to discover many valuable properties in the science, which subsequent speculations gradually improved, and which still continue to this day.

The utility of Geometry extends to almost every art and science. It is by the help of it that astronomers turn their observations to advantage, regulate the duration of time, seasons, years, cycles, and epochas, and measure the distances, motions, and magnitudes of the Heavenly Bodies. It is by it that geographers determine the figure and magnitude of the whole earth, and delineate the extent and bearings of kingdoms, provinces, harbours, &c. It is from this science, too, that architects derive their just dimensions and proportions in the construction of public edifices as well as private houses. It is by the assistance of Geometry that engineers conduct their work for machines of all denominations, as also the situation and plan of towns, and distances of places, and the measure of such things as are only accessible to the sight. It is not only an introduction to Fortification, but highly requisite to every branch of mechanism generally.

On Geometry likewise depends the theory of Music, Optics, Perspective, Drawing, Hydraulics, Pneumatics, &c. &c.

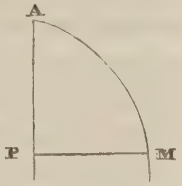
In this work, it must be observed, the various geometrical figures are laid down with simplicity and perspicuity, suitable alike for the artisan, manufacturer, and gentleman ; and, being principally drawn to the full size or scale of the common two-foot rule, are thereby rendered plain to the understanding and easy of imitation. The student will therefore find in this work the best and readiest information in the science of Geometry and Mensuration, which, combined together, are calculated to lead to every relative object, however intricate or remote. For by the methods shown in the divisions of the different figures, the author was enabled to trace out the solutions to the intricate questions of the ancients ; and it is made evidently to appear, that there is no other device existing which would perform similar operations in the same space of time (or, indeed at all), so perfectly correct, both in quantity and formation.

May, 1837.

GEOMETRICAL ILLUSTRATIONS

MATHEMATICALLY DEMONSTRATED.

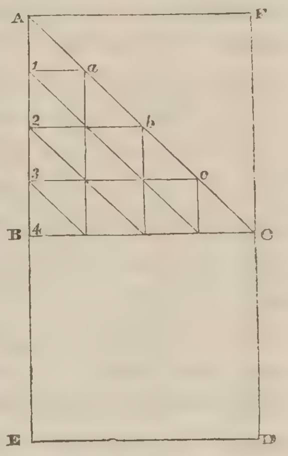
Abscisse, in conic sections, the part AP of the diameter of a curve line, intercepted between the vertex A of that diameter and the point P, where any ordinate or semi-ordinate, MP, to that diameter falls.



From this definition it is evident that there are an infinite number of variable abscisses in the same curve, as well as an infinite number of ordinates.

In the parabola one ordinate has but one abscisse, in an ellipsis it has two, and in curves of the second and third order it may have three or four.

Acceleration denotes the augmentation or increase of motion in accelerated bodies. The term acceleration is chiefly used in speaking of falling bodies, or the tendency of heavy bodies towards the centre of the earth, produced by the power of gravity, which, acting constantly and uniformly upon them, must necessarily cause them to acquire, every instant, a new increase of motion.



Thus, if 1 *a* represent the velocity acquired whilst a body falls through A 1, suppose one minute, then 2 *b* will express the velocity acquired in two minutes, represented by A 2, 3 *c* the velocity acquired in three minutes, or A 3, and BC the velocity acquired whilst the body falls through AB. Now the triangles A 1 *a*, A 2 *b*, A 3 *c*, and ABC represent the spaces described by the falling body in the respective times, A 1, A 2, A 3, and AB, by reason of the uniform action of gravity; but these triangles, being similar, are to each other as the square's homologous sides, A 1, A 2, A 3, and AB, that is, the spaces are to each other as the squares of the times in which they are described. Hence also follows the great law of acceleration, viz. that a falling body, uniformly accelerated, describes in the whole time of its descent just one-half of the space it would have described in the same time with the motion it has acquired at the end of its fall. From what has been said, it is evident that the space described by a falling body, in a series of equal portions of time, will be as the odd numbers 1, 3, 5, 7, &c. See the figure above referred to, where the space described in the time A 1 is represented by the triangle A 1 *a*; whereas the space described in the second portion of time contains three such triangles; that, described in the third portion, five such triangles; and so on. Again, that the spaces described by falling bodies in different times are as the squares of the velocity acquired at the end of its fall. As the spaces represented by the odd numbers 1, 3, 5, 7,

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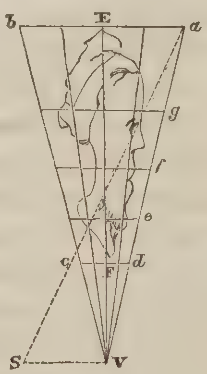
&c., still approach nearer and nearer to equality, so the accelerated motion likewise approaches nearer and nearer to a uniform motion ; and, if the body moves in a resisting medium, the motion will actually become uniform, at a certain distance.

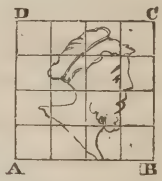
ALTITUDE, one of the three dimensions of a body, being the same with what is otherwise called height.

ALTITUDE of a figure is the nearest distance of its vertex from its base, or the length of a perpendicular let fall from the vertex to the base.

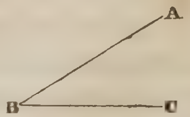
ANALYSIS, the art of discovering the truth or falsehood of a proposition, or its possibility or impossibility. This is done by supposing the proposition, such as it is, true ; and examining what follows from thence until we arrive at some evident truth, or some impossibility, of which the first proposition is a neccessary consequence; and from thence establish the truth or impossibility of that proposition. The analysis of the ancient geometricians consisted in the application of the propositions of Euclid, &c. until they arrived at the truth required. That of the moderns, though not so elegant, must be allowed to be more ready and general. By this last geometrical demonstrations are wonderfully abridged. A number of truths are frequently expressed by a single line, and whole sciences may sometimes be learned in a few hours which otherwise would be scarcely attained in many years.

ANAMORPHOSIS, in perspective and painting, &c. a monstrous projection, or representation of an image on a plane or curved surface, which, beheld at a proper distance, shall appear regular and in proportion. To delineate an anamorphosis upon a plane, draw the square ABCD, and subdivide it into a number of little squares ; in this square, let the image, to be represented deformed, be drawn. Then draw the line *a b* equal to AB, and divide it into the same number of equal parts as the side of prototype, AB. Erect the perpendicular EV in the middle of *a b*, so much the longer as the deformity of the image is to be greater. Draw VS perpendicular to EV, so much the shorter as you would have the image appear more deformed. From each point of division draw straight lines to V, and join the points *a* and S by the right line *a* S. Through the points *d e f g* draw right lines parallel to *a b*, then will *a b c d* be the space in which the monstrous projection is to be delineated : in every small trapezium of the space *a b c d* draw what appears delineated in the correspondent areola of the square ABCD; and thus you will obtain a deformed image, which will appear in just proportion to an eye distant from it the length FV, and raised above its height VS.

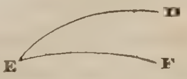


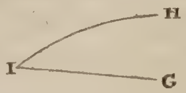


ANGLE, the inclination of two lines meeting each other in a point, and called the legs of the angle. Thus ABC is the angle made by the two lines AB, BC, meeting in the point B, which is the vertex of the angle.

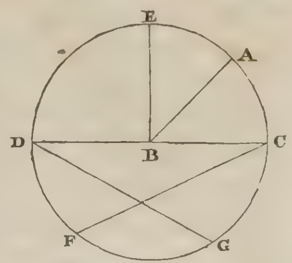


Angles are rectilineal, or right lined, as ABC, above referred to; or they are curvilineal as DEF; or, lastly, formed of a straight line and a curved one, and thence called mixed, as HIG.





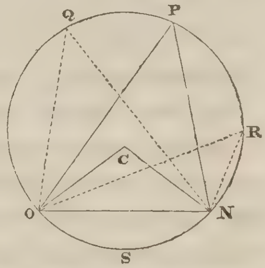
Rectilineal angles, according to the greater or less degree of inclination, are right, acute, or obtuse.



Right angle is that formed between two lines, one of which stands upright, or perpendicularly on the other, inclining no more one way than it does the other. Such is the angle EBC; for, if BC be produced to D, EB will be found to stand upright on DC, or to incline neither way. A right angle is said to be an angle of ninety degrees, because measured by a quadrant of a circle, or $\frac{360}{4}=90$; so that a right angle, or an angle of ninety degrees, is the same thing.

Acute angle, one whose vertex is acute or sharp, being always less than a right angle; such is the angle ABC.

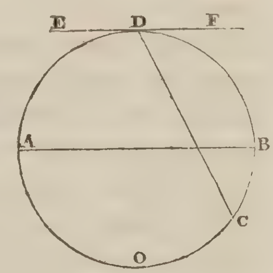
Obtuse angle, one with a blunt or obtuse vertex, as ABD, which is always greater than a right angle. Angles likewise receive other denominations from their different positions, and the relations they bear to the figures they are in and to the lines which form them. Hence,



Angles in a semicircle, those subtended by the diameter of that circle; as, DFC, DGC, which are always right angles.

Angle at the centre, that formed by two radii, or semi-diameters of a circle as, OCN.

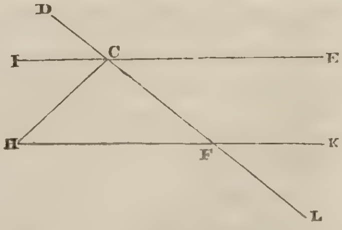
Angle at the circumference, or in a segment, that formed by two chords of a circle meeting at the circumference; such is OPN, which is only half the angle at the centre OCN subtended by the same chord ON, or, which comes to the same thing, it is equal to half the arc OSN. Moreover, all angles in the same segment, and consequently subtended by the same chord ON, as OQN, OPN, ORN, are equal to one another.



Angle of a semicircle, that formed by a diameter and the circumference of a circle, as BAO, which is less than a right angle, and yet greater than any rectilineal-acute one.

Angle of a segment, that which a chord in a circle makes with the tangent at the point of contact; such are the angles EDC, FDC; the former being the angle of the greater segment, and the latter of the lesser segment.

Angle of contact, that which the tangent of a circle forms with its circumference; as, EDA, which is less than any right-lined angle.



Angles are said to be adjacent, or contiguous, which have one leg common to both; as, DGI and DGE, which taken both together are equal to two right angles.

Opposite, or vertical angles, those formed by two lines crossing each other, as the angles DGI, EGF, which are always equal.

An angle is also said to be opposite to the side that subtends it; thus GHF is to the side opposite GF. Again, when one of the sides of a triangle is produced, as from F to K, the external angle GFK is equal to the two internal angles FGH and FHG, which are said to be opposite to it.

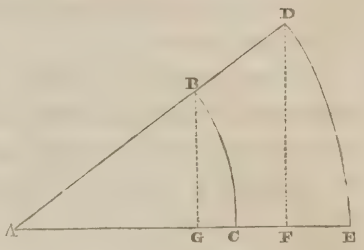
Alternate angles, the internal pairs of acute or obtuse angles, formed by a right line DL cutting two parallel right lines IE, HK: such are EGF, and GFH, both acute and equal; also the obtuse ones IGF and GFK, likewise equal.

Spherical angle, that formed by the intersection of two great circles of the sphere.

Solid angle, that formed by the meeting of three or more plane angles, not being in the same plane, in one point: such is the angle of a die, of a square box, or the like. In regard to solid angles, it has been demonstrated that the plane angles forming them are always less than three hundred and sixty degrees, or four right angles.

ANGULAR, in a general sense, denotes something relating to or having angles.

ARCH, any part of the circumference of a circle, or curved line, lying from one point to another, by which the quantity of the whole circle or line, or some other thing sought after, may be obtained. If the arch BC contains the same number of degrees as the arch DE, or, if the radius AB is to the radius AD as the arch BC to the arch DE, then these two arches are similar.



ARCHES EQUAL, those which contain the same number of degrees, and whose radii are equal.

ARCH, in architecture, a concave building, with a mould bent in the form of a curve, erected to support some structure. Arches are either circular, elliptical, or strait, as they are improperly called by workmen. Circular arches are of three kinds: First, Semicircular, which have their centre in the middle of a line drawn between the feet of the arch. Secondly, Scheme, or Skene, which are less than a semicircle, containing some ninety, and some seventy degrees. Thirdly. Arches of the third and fourth point, consisting of two arches of a circle meeting in an angle at the top, being drawn from the division of a chord into three or more parts at pleasure.

Elliptical arches consist of a semi-ellipsis, and have commonly a key stone and imposts: they are usually described on three centres. Straight arches are those used over doors and windows, having plain straight edges, both upper and under, which are parallel, but both the ends and joints point towards a centre.

Arch is also used to denote the interval between two piers of a bridge.

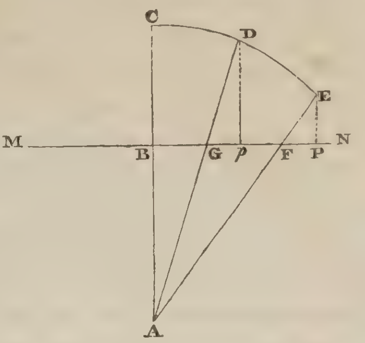
AREA, denotes the superficial content of any figure: thus, if we suppose a parallelogram six inches long, and four broad, its area will be $6 \times 4=24$ square inches.

ART, a system of rules serving to facilitate the performance of certain actions, in which sense it stands opposed to science, or a system of merely speculative principles. Arts are commonly divided into liberal and mechanical, the former comprehending poetry, painting, sculpture, architecture, &c., and the latter the whole body of mechanical trades; as, carpenters', engineers', masons' work, &c.

ARTICLE, sometimes called decade, denotes a number justly divisible into ten parts, as the numbers 10, 20, 30, &c.

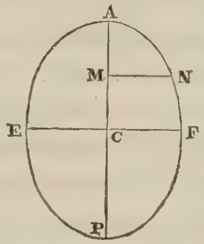
ASSYMPTOTE, a line which continually approaches nearer to another line, but which, though continued infinitely, will never meet it: of these there are many kinds. In strictness, however, the term assymptotes is appropriated to right lines which approach nearer and nearer.

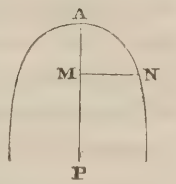
nearer to some curve of which they are said to be assymptotes; but, if they and their curve are indefinitely continued, they will never meet.

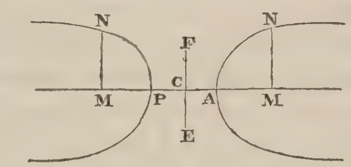


The nature of an assymptote will be easily conceived from considering the assymptote of the chonchoid; for if C DE be a part of the curve of a conchoid, and A its pole, and the right lines MN be so drawn that the parts, BC, GD, FE, of right lines drawn from the pole A be equal to each other, then the line MN will be the assymptote of the curve; because the perpendicular D p is shorter than BC, and EP shorter than D p, and so on; and the points E and p can never coincide.

Axiom is an established principle in some art or science. Thus it is an established axiom in physics, that nature does nothing in vain; so it is in geometry that if to two equal things you add equals the sums will be equal. It is an axiom in optics that the angle of incidence is equal to the angle of reflection, &c. In which sense, too, the general laws of motion are called axioms. Whence it may be observed, that these particular axioms are but deductions from certain hypothesis.





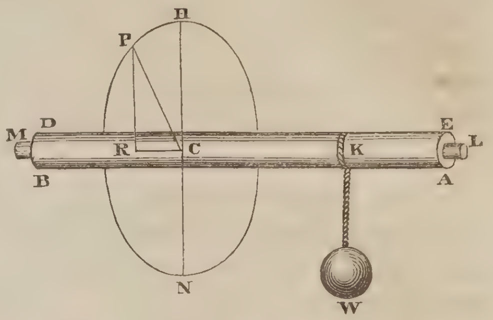


Axis, in conic sections, a right line dividing the section into two equal parts, and cutting all its ordinates at right angles. Thus if A P be drawn so as to cut the ordinate MN, at right angles, and divide the section into two equal parts, then is the line AP the axis of the section. The transverse, first, or principal axis of an ellipsis or hyperbola is the axis AP, which, in the ellipsis, is the longest, and in the hyperbola cuts the curves in the points A and P. The conjugate, or second axis of an ellipsis, is the line EF drawn through the centre C, parallel to the ordinate MN, and perpendicular to the transverse axis AP, being the shorter of the two, and terminated by the curve. The conjugate axis of the hyperbola is the right line EF, drawn through the centre C, parallel to the ordinates MN, and perpendicular to the transverse axis AP. This axis, though no more than infinite, is of a determinate length, and may be found by this proportion, as $AM \times PM : AP^2 :: MN^2 : EF^2$. The axis of a parabola is of an indeterminable length. The axis of the ellipsis is determinate. In the ellipsis and hyperbola there are two axis and no more, and in the parabola only one.

Axis, in mechanics. The axis of a balance is that about which it moves, or rather turns. Axis of oscillation is a right line parallel to the horizon, passing through the centre about which a pendulum vibrates.

Axis, in peritrochio, one of the five mechanical powers, consisting of a peritrochium, or wheel, concentric with the base of a cylinder, and movable together with it about its axis. The power is applied at the circumference of the wheel, and the weight is raised by a rope that is gathered up on the axis or cylinder while the machine turns round.

The power may be conceived as applied at the extremity of the arm of a lever equal to the radius of the wheel, and the weight as applied at the extremity of a lever, equal to the radius of the axis; only those arms do not meet at one centre of motion, as in the lever but in place of this centre there is an axis of motion, viz. the axis of the whole machine. But, as this can produce no difference, it follows that the power and weight are *in equilibrio* when they are to each other inversely as the distance of their directions from the axis of the engine; or when the power is to the weight as the radius of the roller to the radius of the wheel, the power being supposed to act in a perpendicular to this radius.



But, if the power act obliquely to the radius, substitute a perpendicular from the axis on the direction of the power, in the place of the radius. Thus, if ABDE represent the cylindrical roller, HPN the wheel, LM the axis, or right line, upon which the whole engine turns, Q the point of the surface of the roller where the weight W is applied, P the point where the power is applied, KQ the radius of the roller, CP the radius of the wheel, then, if the power P act with a direction perpendicular to CP, the power and weight will sustain each other, when P is to W as KQ is to CP or CH. But, if the power act in any other direction PR, let CR be perpendicular from C the centre of the wheel on that direction, then P and W will sustain each other, when P is to W as KQ to CR; because, in this case, as power P has the same effect as if it was applied to the point R of its direction, acting in a right line perpendicular to CR.

The use of this machine is to raise weight to a greater height than the lever can do; because the wheel is capable of being turned several times round, which the lever is not, and also to communicate motion from one part of a machine to another.

Axis, in optics, is that ray among all others that are sent to the eye which falls perpendicularly upon it, and which consequently passes through the centre of the eye. Common or mean axis is a right line drawn from the point of concourse of the two optic nerves through the middle of the right line which joins the extremity of the same optic nerves.

Axis of a vessel is an imaginary light line, passing through the middle of it perpendicularly to its base, and equally distant from its sides.

Base, the lowest side of the perimeter of a figure: thus, the base of a triangle may be said of any of its sides, but more properly of the lowest, or that which is parallel to the horizon. In rectangled triangles, the base is properly that side opposite to the right angle.

Base of a solid figure the lowest side, or that on which it stands; and if the solid has two opposite parallel plane sides, and one of them is the base, then the other is called the base also.

Base of a conic section is a right line in the hyperbola and parabola, arising from the common intersection of the secant plane and the base of the cone.

Base line, in perspective, the common section of a picture, and the geometrical plane.

Bimedial. If two medial lines, as AB, and BC, commensurable only in power, containing a rational rectangle, are compounded, the whole line AC will be irrational, and is called a first bimedial line.

Bisection, the division of a line, angle, &c. into two equal parts.

Body is otherwise called a solid. The regular bodies, or those which have all their angles and sides similar and equal, are five, viz., the tetraehedron, octahedron, dodecahedron, icosahedron, and the cube.

Breadth, one of the three dimensions of bodies which multiplied into their length constitute a surface.

Centre of a circle, a point in the middle of a circle, or circular figure, from which all lines drawn to the circumference are equal.

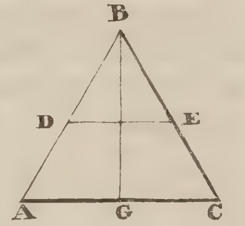
Centre of a conic section, a point where the diameters intersect each other.

Centre in the ellipsis, the point is within the figure, and in the hyperbola without.

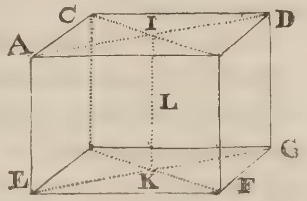
Centre of an ellipsis, the point where the transverse and conjugate diameters intersect each other.

Centre of gravity, that point about which all the parts of a body, in any situation, invariably balance other other.

The centre of gravity in regular, uniform, and homogeneal bodies, as squares, circles, &c., is the middle point in a line connecting any two opposite points, or angles. Therefore, if such line be bisected, the point of section will be the centre of gravity. To find the centre of gravity of a triangle, bisect the base AC of the triangle ABC ; it will also bisect every other line, DE, drawn parallel to the base ; consequently, the centre of gravity of the triangle will be found somewhere in the line BG.



To find the centre of gravity in a parallelogram and parallelopiped, draw the diagonal AD and EG, likewise CB and HF ; since each diagonal AD and CB divide the parallelogram ACDB into equal parts, each passes through the centre of gravity; consequently the point of intersection, I, must be the centre of gravity of the parallelogram. In like manner, since both the plane CBFH and ADGE divide the parallelopiped into two equal parts, each passes through its centre of gravity, so that the common intersection IK is the diameter of gravity, the middle thereof, L, is the centre. From similar directions may the centre of gravity be found in prisms, cylinders, &c., it being the middle point of the right line that joins the centre of gravity of their opposite bases. The centre of gravity of a parabola is found as in the triangle and cone.



Centre of the hyperbola, a point in the middle of the transverse axis.

Centre of magnitude of any homogeneal body, the same with the centre of gravity.

Centre of motion, that point which remains at rest, while all the other parts of a body move about it. This is the same in uniform bodies of the same matter throughout, as the centre of gravity.

Centre of percussion, in a moving body, that point wherein the percutient force is greatest, or that point with which, if the body strikes against any obstacle, no shock shall be felt at the point of suspension. The centre of percussion, when the percutient body revolves round a fixed point, is the same with the centre of oscillation, and consequently may be determined by the same rule, which is as follows, viz. if several bodies be fixed to an inflexible rod suspended upon a point, and each body be multiplied by the square of its distance from the point of suspension, and then each body be multiplied by its distance from the same point, and all the former products when added together be divided by all the latter products added together, the quotient which shall arise from thence will be the distance of the centre of oscillation of these bodies from the said point. Hence a stick of a cylindrical figure, supposing the centre of motion at the hand, will strike the greatest blow at a distance of two-thirds of its length from the hand. The centre of percussion is the same with the centre of gravity, if all the parts of the percutient body be carried with a parallel motion, or with the same celerity. For the momenta are the facta of the weights into the celerities, therefore to multiply equiponderating bodies by the same velocity is the same thing as to take equimultiples of them. But the equimultiples of equiponderating bodies themselves equiponderate; therefore equivalent momenta are disposed about the centre of gravity, and, consequently, the centre of gravity in this case will coincide with the centre of percussion, and what is shown of the one will hold of the other.

Centre of conversion may be explained thus. If a stick be laid on stagnant water, and drawn by a thread fastened to it, so that the thread always makes the same angle with the stick, viz. a right angle, the stick will be found to turn on one of its points, which will be immovable, and this point is called the point of conversion. This effect arises from the resistance of the fluid; but the great question consists in knowing in what point the centre of conversion is found. This has been calculated with a great deal of exactness, for if the stick be drawn by one extremity in a straight line divided into twenty parts, the centre of conversion will be nearly on the thirteenth, reckoning from the thread. If it be not a line, but a surface or solid, there will be some change in the situation of the centre of conversion, according to the nature of the surface or solid.

Centre of a parallelogram, or polygon, the point in which all its diagonals intersect.

Centre of a sphere, a point in the middle from which all lines drawn to the surface are equal.

CENTRAL FORCES, the powers which cause a moving body to tend towards, or recede from, the centre of motion.

CENTRIFUGAL FORCE, that force by which all bodies that move round any other body in a curve endeavour to fly off from the axis of their motion in a tangent to the periphery of the curve, and that in every point of it. This force is always proportional to the circumference of the curve in which the revolving body is carried round. The centrifugal force of any body is to the centripetal as the square of the arch which a body describes in a given time, divided by the diameter, to the space through which a heavy body moves in falling from a place where it was at rest in the same time. If any body swim in a medium heavier than itself, the centrifugal force is the difference between the specific weight of the medium and the floating body. All moving bodies endeavour a rectilinear motion, because it is the easiest, shortest, and most simple. Whenever, therefore, they move in any curve, there must be something that draws them from the rectilinear motion, and detains them in their orbits; and, were that force to cease, the moving body would go straight off in a tangent to the curve in that very point, and would get further and further from the focus, or centre of its curvilinear motion. It may be that, in a curve, where the force of gravity in the describing body is continually variable, the centrifugal force may also continually vary in the same manner, and so that one may also supply the defect, or abate for the excess of the other, consequently the effect be everywhere equal to the absolute gravity of the revolving body.

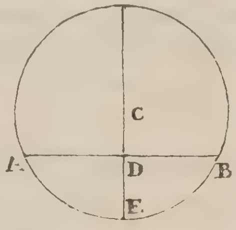
Centripetal force, that force by which a body is every where impelled, or any how tends towards some point as a centre. Such is gravity, or that force whereby bodies tend towards the centre of the earth; magnetical attraction, by which the loadstone draws iron; and that force, whatever it be, whereby the planets are continually drawn back from right lined motions and made to move in curves. The greater the quantity of matter in any body is the greater will be its centripetal force, all things else alike. If a body laid upon a plane, revolve at the same time and about the same centre with that plane, and so describe a circle; and if the centripetal force, wherewith the body is drawn every moment towards the centre, should cease to act, and the plane should continue to move with the same velocity, the body will begin to recede from the centre about which the plane moved.

Centrum, the same with centre.

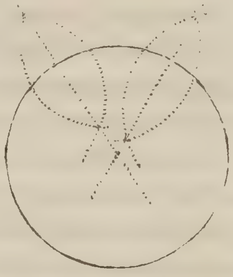
Characters in geometry and trigonometry, viz. ‖, parallelism; △, triangle; □, square; ▯, ▭, rectangles; ○, circle; $\underline{\underline{\vee}}$, equi-angular or similar; $\underline{\perp}$, equilateral; ∠, an angle; ∟, a right angle; ⊥, perpendicular.

Chord of the circle, a right line drawn from one part of an arch of a circle to the other. Hence the chord of an arch is a right line joining the extremes of that arch; thus, AB is the chord of the arch AEB. It is demonstrated that the radius CE, bisecting the chord BA in D, does also bisect the arch in E, and is perpendicular to the chord AB.

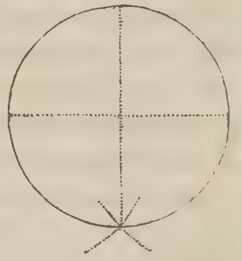
From hence may be deduced the following problems:—



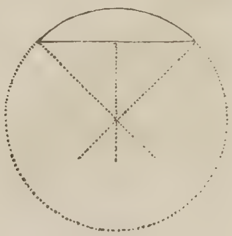
First, To make a circle pass through any three given points, placed indiscriminately, and not in a right line.



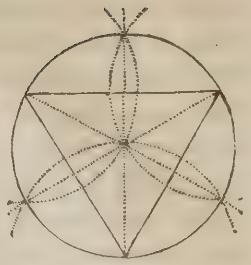
Secondly, To find the centre of any circle.



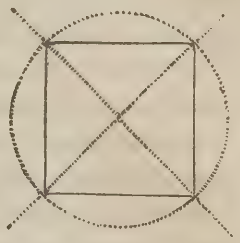
Thirdly, To complete a circle from an arch given.



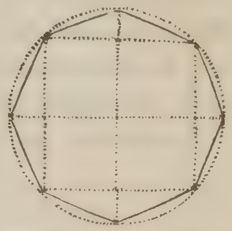
Fourthly, To describe a circle about any triangle given.



Fifthly, To describe a square within a given circle.



Sixthly, To describe an octagon with a given circle.



Cipher, one of the Arabic characters, or figures, used in computation, formed thus 0. A cipher of itself signifies nothing; but when placed to the left of other characters, in whole numbers, augments their value ten times; and, when placed to the right hand in decimal arithmetic, lessens the value of each figure in the same proportion.

Cipher denotes likewise certain characters disguised and varied, used in writing letters that contain some secret not to be understood but by those between whom the cipher is agreed on: for instance the word cut may be formed by the circle and square; viz. take as much from the circle as will unite two sides of the square, and transfix the other two sides as CUT.

Circle, a plane figure, comprehended by a single curved line, called its circumference, to which right lines, drawn from a point in the middle, called the centre, are equal to each other. The area of the circle was found by multiplying the circumference by the fourth part of the diameter, or half the circumference by half the diameter; for every circle may be conceived to be a polygon of an infinite number of sides, and the semi-diameter must be equal to the perpendicular of such a polygon, and the circumference of the circle equal to the periphery of the polygon; therefore, half the circumference, multiplied by half the diameter, give the area of the circle. Circles, and similar figures inscribed in them, are always as the squares of the diameters; so that they are in duplicate ratio of their diameters, and consequently of their radii. A circle is equal to a triangle, the base of which is equal to the periphery, and its altitude to its radius: circles, therefore, are in a ratio compounded of the peripheries and the radii.

To describe a circle through three given points, see Chord.

For the divisions of the Circle, see the plate; and for the quadrature of the circle, or the manner of making a square whose surface is perfectly and geometrically equal to that of a circle, is a problem that has employed the geometricians of all ages. "Charles the Fifth offered a reward of one hundred thousand crowns to the person who should solve this celebrated problem, and the States of Holland have also proposed a similar reward for the same purpose." See the plate.

Circle of curvature, a circle the curvature of which is equal to that of a certain curve at a given point.

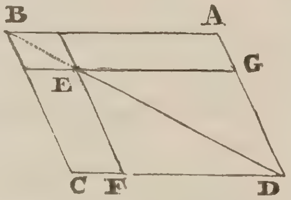
CIRCULAR, in a general sense, any thing that is described or moved in a round, as the circumference of a circle, or surface of a globe or sphere. The circular form is of all others the best disposed for motion, and the most capacious.

CIRCUMFERENCE, in a general sense, denotes the line or lines bounding a plane figure. However, it is generally used, in a more limited sense, for the curved line which bounds a circle, and otherwise called a periphery, being the boundary of a right lined figure expressed by the term perimeter. Any part of the circumference is called an arch, and a right line drawn from one extreme of the arch to the other is called a chord.

COMMENSURABLE, an appellation given to such quantities as are measured by one and the same common measure; thus, if the line A ÷÷÷÷÷ *a* be equal to 8, and the line B ÷÷÷÷÷ *b* equal to 4 inches, these two lines will be commensurable, since the same common measure of two measures them both.

COMPLEMENT is what remains of a quadrant of a circle, or of 90°, after any certain arch has been taken away from it. Thus, if the arch taken away be 40°, its complement is 50°, because $50 + 40 = 90$.

COMPLEMENTS, in a parallelogram, are the two smaller parallelograms GAE, FCE, made by drawing two right lines GE and FE through the point E in the diagonal, and parallel to the sides AB BC of the parallelogram ABCD. In every parallelogram these complements are equal.



COMPOUND MOTION, that which is affected by the concurring action of several different powers. Thus, if one power act in the direction of, and with a force proportional to, the end of a parallelogram, and another act in the direction of, and with a force proportional to, its side, the compound motion will be in the direction of, and proportional to, the diagonal of the parallelogram.

CONCAVE, the inner surface of hollow spherical bodies.

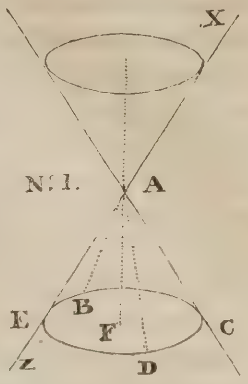
CONCAVITY, the whole space included within a concave surface, or the inner bend of a curved line.

CONCENTRIC is that body that has the same common centre with another, it stands in opposition to eccentric. Concentric is also used in speaking of round bodies and figures, or circular and elliptical ones; but may likewise be used for polygons drawn parallel to each other, upon the same centre.

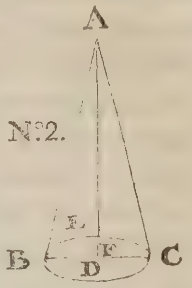
Cone, a solid figure, having a circle for its base, and its top terminated in a point or vertex.

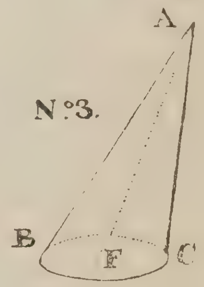
A Cone may be conceived to be generated in the following manner:—

Take an immovable point A, elevated above the plane of a circle BCDE, and suppose a straight line XZ, drawn through the point and extended both ways from it to an indefinite length, to be carried round the circle, continually touching its circumference and remaining fixed to the immovable point, the line by this motion will describe two conic surfaces, which are vertical or opposite, having their common vertex at the immovable point.

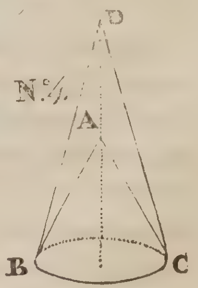

N°1.

The solid contained within this conic surface between the immovable point A and the circumference of the circle BCDE is a cone. The immovable point A is the vertex, the circle BCDE is the base, and a straight line AF drawn from the vertex to the centre of the base is the axis of the cone. All straight lines drawn from the vertex to the circumference of the base, as AB, AC, AD, AE, &c., are sides of the cone. If the axis of a cone be perpendicular to its base it is called a right or proper cone, as in No. 2. If the axis be inclined to the base, it is called a scalinous or oblique cone, such as No. 3, and a right cone is always understood when the contrary is not expressed.


N°2.

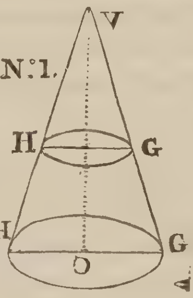

N°3.

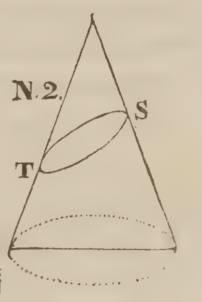
Euclid, in his eleventh book, gives a definition of a cone that is not general, it being only of a right cone. He says, a cone is produced by the revolution of the plane of a right angled triangle, about the perpendicular axis remaining at rest. If this axis be greater than half the base, the solid produced is an acute angled cone; if less, it is an obtuse angled cone; and if equal, a right angled cone. Thus the cone BAC, No. 4, is less acute than the cone BDC, because the angle BDF is less than the angle BAF.


N°4.

Cone. Cones and pyramids, having the same bases and altitudes, are equal to each other. It is to be observed that every triangular prism may be divided into three equal pyramids, and therefore a triangular pyramid is one third of a prism standing on the same base, having the same altitude. Every multangular body may be resolved into triangular ones. A pyramid is the third part of a prism, standing upon the same base, having the same altitude; and as a cone may be esteemed an infinite angular pyramid, and a cylinder an infinite angular prism, a a cone is a third part of a cylinder, which has the same base and altitude. The method of measuring the solidity and surface of a cone and pyramid is, Find the solidity of a prism or cylinder having the same base with the cone or pyramid, which found divide by three: the quotient will be the solidity of the cone or pyramid. Or the solidity of any cone is equal to the area of the base multiplied into one-third part of its altitude. As for the surfaces, that of a right cone, not taking in the base, is equal to a triangle whose base is the periphery and altitude the side of the cone; therefore, the surface of a right cone is obtained by multiplying the periphery of the base into half of the side, and adding the product to that of the base. The altitudes of similar cones are as the radii of the bases, and form the same angle with them. Cones are to one another in a ratio compounded of their bases and altitudes. Similar cones are in a triplicate ratio of their homologous sides, and likewise of their altitudes. Of all cones standing upon the same base, and having the same altitude, the superficies of that which is most oblique is greatest, and the superficies of the right cone the least.

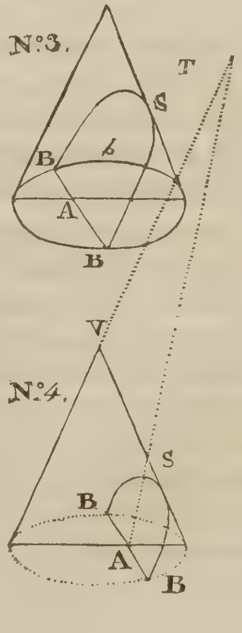
Conic Sections, curves formed from the section of a cone by a plane. The curves that generally pass under the name of conic sections are three, viz. the ellipsis, parabola, and hyperbola. Although the triangle and circle are formed from the sections of a cone, yet they are not always considered in that capacity.





If a right cone be cut directly through its axis, the plane or superficies of that section will be a plain isosceles triangle, as HVG; for HV and VG, being the sides of the cone, will be the sides of the triangle. HG, the diameter of the cone's base, will also be the base of the triangle, and its axis, VC, will be the height or altitudinal line. If the cone be cut by a right line parallel to its base, the plane of that section will be a circle, because the base of the cone is a circle. Such is HG.

If a cone be cut by a right line that cuts both its sides, but not parallel to its base, as TS, No. 2, the plane of that section will be an ellipsis, commonly called an oval; that is an oblong or imperfect circle, having several diameters and two particular centres.



If a cone be cut into two parts by a right line parallel to one of its sides, as SA, No. 3, the plane of that section, namely, S *b* BA *b*, is called a parabola.

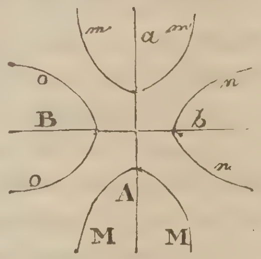
If a cone be cut by a right line parallel to its axis, as SA, No 4, in such a manner that the intersecting line when continued through one side of the cone, as at S, will meet with the other side if it be continued beyond the vertex V, as at T, then is the plane of that section, namely, SB, AB, called an hyperbola.

These five sections, namely, the triangle, circle, ellipsis, parabola, and hyperbola, are all the planes that can be produced from a cone. But of them the last three are only sections, so called by ancient and modern geometricians.

From the production of these sections, it may be observed how one section degenerates into another. For an ellipsis being that plane of any section of the cone which is between the circle and parabola, it will be easy to conceive there may be great variety of ellipses produced from the same cone; and, when the section happens to be exactly parallel to one side of the cone, then the ellipsis degenerates into a parabola. Now the parabola, being a section whose plane is always exactly parallel to the side of the cone, cannot vary as the ellipsis; for, as soon as it begins to move out of the position of being parallel to the side of the cone, it degenerates either into an ellipsis or hyperbola. That is, if the section inclines towards the plane of the cone's base it becomes an ellipsis, but if it incline towards the cone's vertex it then becomes a hyperbola, which is the plane of any section that falls between the parabola and the triangle; therefore there may be as many varieties of hyperbolas produced from the same cone as there may be ellipses. A circle may change into an ellipsis, the ellipsis into a parabola, the parabola into a hyperbola, and the hyperbola into a plain isosceles triangle; and the centre of the circle, which is its focus, divides itself into two focuses, as soon as the circle begins to degenerate into an ellipsis; but, when the ellipsis changes into a parabola, one end of it opens and one of its foci vanishes, and the remaining go along with the parabola, when it degenerates to a hyperbola. When the hyperbola degenerates into a plain isosceles triangle, this focus becomes the vertical point of the triangle, namely, the vertex of the cone; so that the centre of the cone's base may be said to pass gradually through all the sections until it arrive at the vertex of the cone, for the diameter of a circle being the right line that passes through its centre or focus, from which all other right lines drawn within the circle are regulated.

Conjugate diameter, or axis of an ellipsis, the shortest of the two diameters, or that bisecting the transverse axis.

Conjugate hyperbolas. If there be two opposite hyperbolas, AM, *am*, whose principal axis is the line A *a*, and the conjugate axis the line B *b*—if there be two other hyperbolas whose principal axis is the line B *b*, and conjugate the line A *a*—then those four hyperbolas are called conjugate hyperbolas, the two former being conjugates to the latter.



Conoid, a solid body, generated by the revolution of a conic section about its axis.

Elliptical conoid, a solid formed by the revolution of an ellipsis about one of its diameters, and more generally called a spheroid.

Hyperbolical conoid is generated by the revolution of a hyperbola about its axis.

Parabolical conoid is generated by the revolution of a parabola about its axis.

Construction is the drawing lines, &c., as are necessary for the making any demonstration appear more plain and undeniable.

Contact is when one line, plane, or body is made to touch another, and the parts that do thus touch are called the points or places of contact. The contact of two spherical bodies, and of a tangent with the circumference of a circle, is only in one point.

Content, the area or quantity of matter or space included within certain bounds. The content of a ton of round timber is 43 feet. A load of hewn timber is 50 feet. In a foot of timber are contained 1728 cubic or square inches, and as often as 1728 are contained in a piece of timber, be it round or square, so many feet of timber are contained in the piece.

Contiguity is when the surface of one body touches another.

Contiguous angles are such as have one leg common to each angle, and are not unfrequently called adjoining angles, in contradistinction to those produced by continuing their legs through the point of contact, which are called opposite or vertical angles.

The sum of any two contiguous angles is always equal to two right angles.

Contingent lines are those which are casual and uncertain.

Contractile force, that property or power inherent in certain bodies after being extended they are enabled to draw themselves up again to their original dimensions.

Converging or **Convergent lines** are such as continually approach nearer to one another, or whose distance become still less and less. These are opposed to divergent lines, the distance of which becomes continually greater. Those lines which converge one way diverge the other.

Converging hyperbola is one whose concave legs bend in towards one another, and continue both the same way.

Converse. One proposition is called the converse of another, when, after a conclusion is drawn from something supposed in the converse proposition, that conclusion is supposed; and then that which in the other was supposed is now drawn as a conclusion from it. Thus, when two sides of a triangle are equal, the angles under these sides are equal; and on the converse, if these angles are equal, the two sides are equal.

Convex, an appellation given to the exterior surface of globular bodies, in opposition to the hollow inner surface of such bodies as are called concave.

Corollary is a useful consequence drawn from something already advanced or demonstrated: thus, it being demonstrated that a triangle which has two equal sides has also two angles equal, this corollary will follow, that a triangle which has three sides equal has all its three angles equal.

Co-sine, the sine of an arch, which is the complement of another to 90 degrees.

Crown is a plane ring included between two concentric perimeters, and is generated by the motion of some part of a right line round a centre, the moving part not being contiguous to it.

Cube, a solid body consisting of six equal square sides. The solidity of any cube is found by multiplying the superficial area of one of the sides by the height.

Curvature of a line is the peculiar manner of its bending or flexure by which it becomes a curve of such and such peculiar properties.

Any two arches of curved lines touch each other when the same right line is the tangent of both at the same point; but, when they are applied upon each other in this manner, they never perfectly coincide, unless they are similar arches of equal and similar figures. And the curvature of lines admit of indefinite variety; because the curvature is uniform in a given circle, and may be varied at pleasure in them, by increasing or decreasing their diameters. The curvature of circles serve for measuring other lines. Of all the circles that touch a curve in any given point, that is said to have the same curvature which touches it so closely that no circle can be drawn through the point of contact between them; and their circles are called the circle of curvature, its centre the centre of curvature, and its semi-diameter the ray of curvature belonging to the point of contact. As in all figures, rectilinear ones excepted, the position of the tangent is continually varying, so the curvature is continually varying in all curvilinear figures, the circle only excepted. As the curve is separated from its tangent by its curvature, so is it separated from the circle curvature in consequence of the increase or decrease of its curvature:

and as its curvature is greater or less, according as it is more or less inflected from the tangent, so the variation of curvature is greater or less, according as it is more or less separated from the circle of curvature. When any two lines touch each other in such a manner that no circle can pass between them, they must have the same curvature; for the circle that touches one so closely that no circle can pass between them must touch the other in the same manner. And it can be made appear that circles may touch curved lines in this manner, that there may be indefinite degrees of more or less intimate contact between the curve and the circle of curvature, and that a conic section may be described that shall have the same curvature with a given line at a given point, and the same variation of a curvature, or a contact of the same kind with the circle of curvature. The rays of curvature of similar arches, in similar figures, are in the same ratio as any homologous lines of these figures, and the variation of curvature the same.

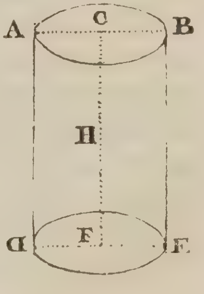
CURVE, a line which, running on continually in all directions, may be cut by one right line in a variety of points. Curves are divided into geometrical and transcendental. Geometrical curves are those whose ordinates and abscisses being right lines the nature thereof can be expressed by a finite equation having those ordinates and abscisses in it. Transcendental curve is such as when expressed by an equation, one of the terms thereof is a variable quantity. Geometrical lines or curves are divided into orders, according to the number of dimensions of the equation expressing the relation between the ordinates and abscisses, or according to the number of points with which they may be cut by a right line. A curve of the first kind (for a right line is not to be reckoned among curves) is the same with a line of the second order, and a curve of the second order the same as a line of the third, and a line of an infinite order is that which a right line can cut into an infinite number of points, such as a spiral, quadratrix, cycloid, the figures of the sines, tangents, secants, and every line which is generated by the infinite revolutions of a circle or wheel.

The curves of the second order have parts and properties similar to the first. As the conic sections have diameters and axes, the lines cut by these are called ordinates, and the intersection of the curve and diameter, the vertex; so in curves of the second order any two parallel lines being drawn so as to meet the curve in three points, a right line cutting these parallels so that the sum of the two parts between the secant and the curve on one side is equal to the third part terminated by the curve on the other side, will cut in the same manner all other lines parallel to these, and meet the curve in three parts, so that the sum of the two parts on one side will be still equal to the third part on the other side.

These three parts, therefore, thus equal, may be called ordinates or applicates, the secant may be styled the diameter, the intersection of the diameter and the curve the vertex, and the point of concourse of any two diameters the centre. And, if the diameter be vertical to the ordinates, it may be called axis, and that point where all the diameters terminate the general centre. Again, as a hyperbola of the first order has two assymptotes, that of the second three, that of the third, four, &c., and as the parts of any right line lying between the conic hyperbola and its two assymptotes are every where equal, so in the hyperbola of the second order, if any right line be drawn cutting both the curve and its three assymptotes in three points, the sum of the two parts of that right line being drawn the same way from any two assymptotes to two points of the curve will be equal to a third part drawn a contrary way from the third assymptote to a third point of the curve. As in conic sections not parabolical the square of the ordinate; that is, the rectangle under the ordinates drawn to contrary sides of the diameter, is to the rectangle of the parts of the diameter which are terminated at the vertices of the ellipsis or by hyperbola, so, in non-parabolic curves of the second order, a parallelopiped under the three ordinates is to a parallelopiped under the parts of the diameter, terminated at the ordinates, and the three vertices of the figure in a certain given ratio, in which ratio, if you take three right lines situated at the three parts of the diameter between the vertices of the figure, one answering to another, then these three right lines may be called the right lines of the figure, and the parts of the diameter between the vertices the transverse lines. And as in the conic parabola, having to one and the same diameter but only one vertex, the rectangle under the ordinates is equal to that under the part of the diameter cut off between the ordinates and the vertex and the right line, so in curves of the second order, which have but two vertices to the same diameter, the parallelopiped under three ordinates is equal to the parallelopiped under two parts of the diameter cut off between the ordinates and those two vertices and a given right line. For as, in conic sections, when two parallels terminated on each side of the curve are cut by two other parallels terminated on each by the curve, the first by the third and the second by the fourth, as the rectangle under the parts of the first is to the rectangle under the parts of the third, as the rectangle under the parts of the second is to that under the parts of the fourth, so when four such right lines occur in a curve of the second kind, each in three points, then shall the parallelopiped under the parts of the first line be to that under the parts of the third, as the parallelopiped under the parts of the second line to that under the parts of the fourth. Lastly, the legs of curves, both of the first, second, and higher kinds, are either of the parabolic or hyperbolic kind, a hyperbolic leg being that which approaches infinitely towards some assymptote, a parabolic that which has no assymptote. These legs are best distinguished by their tangents; for, if the point of contact goes off to an infinite distance, the tangent of the hyperbolic leg will coincide with the assymptote, and the parabolic leg recede infinitely and vanish. The assymptote, therefore, of any leg is found by seeking the tangent of it to a point infinitely distant; and the bearing of an infinite leg is found by seeking the position of a right line parallel to the tangent when the point of contact is infinitely remote, for this line tends the same way towards which the infinite leg is directed.

Cycloid, a curve of the transcendental kind. Upon this curve the doctrine of pendulums and time measuring instruments, in a great measure, depends.

Cylinder, a solid body, supposed to be generated by the rotation of a parallelogram, as CBEF, about one of its sides CF. If the generating parallelogram be rectangular, as CBEF, the cylinder it produces will be a right cylinder, that is, it will have its axis perpendicular to its base. If the parallelogram be a rhombus or rhomboides, the cylinder will be oblique or scalenous. The section of every cylinder by a plane oblique to its base is an ellipsis. The superficies of a cylinder is equal to the periphery of the base multiplied into the length of its side. The solidity of a cylinder is equal to the area of its base, multiplied into its altitude. Cylinders of the same base and standing between the same parallels are equal. Every cylinder is to a spheroid inscribed in it as three to two. If the altitudes of two cylinders be equal to the diameters of their bases, those cylinders are to one another as the cubes of the diameters of their bases. To find a circle equal to the surface of a cylinder we have this theorem:—The surface of a cylinder is equal to a circle whose



radius is a mean proportional between the diameter and height of the cylinder. The diameter of a sphere and altitude of a cylinder equal thereto being given to find the diameter of the cylinder, the theorem is the square of the diameter of the sphere is to the square of the diameter of the cylinder equal to it, nearly, as triple the altitude of the cylinder to double the diameter of the sphere.

CYLINDRICAL, that body which is in the form of or resembling a cylinder.

CYLINDROID, a solid body approaching to the figure of a cylinder, but differing from it in some respects, as having the bases elliptical, but parallel and equal.

DATA, a term for such things or quantities as are given or known, in order to find other things thereby that are unknown. The word data is used for such spaces, lines, and angles as are given in magnitude, or to which others can be assigned equal.

DECAGON, a plane figure with ten sides and ten angles: it is called a regular decagon when all the sides and angles are equal.

DEDUCTION, a subtracting or retrenching a smaller quantity from a greater.

DEFINITION, the elucidating one method by several other not synonymous methods.

DEFINITIVE, a term applied to the termination of a question.

DEGREE, a division of a circle, including the three hundred and sixtieth part of the circumference.

DEMONSTRATION, evident truths or propositions already established.

DEPTH, the same as altitude, although we only use the term depth to denote how much one body or part of a body is below another.

DESCENT is the tendency of a body from a higher to a lower place. All bodies, unless otherwise determined by a force superior to their gravity, descend towards the centre of the earth. Heavy bodies, meeting with no resistance, descend with a uniformly accelerated motion, for the laws of which see Acceleration.

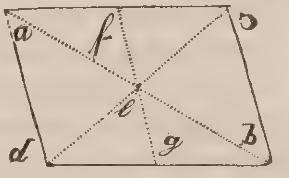
All bodies near the surface of the earth descend perpendicularly at the rate of sixteen feet one inch in a second of time. The velocity of a body descending in an inclined plane at the end of any given time is to the velocity that it would acquire by descending perpendicularly in the same time as the altitude of the inclined plane is to its length. The last velocity acquired by the direct descent is to the last velocity acquired in the same time by the oblique descent as the absolute gravity is to the relative gravity of the descending body. The line described by the direct descent is to the line described in the same time by the oblique descent as the length of the oblique plane is to its height. If the line described by the direct descent be to the line described by the oblique descent as the height of the inclined plane to its length, then the times of descent will also be in the same proportion, and the last velocities equal. The last velocities acquired upon several inclined planes of the same height, however different in length, are equal. The time of any oblique descent through any chord of a circle drawn from its lowest point is equal to the time of a direct descent through the diameter of that circle. The last acquired velocities of a body, descending to the lowest point of a given circle through different chords, are as those chords. The time of the descent of a body in any arch of a semi-cycloid is equal to the time of its descent through any other arch, whether longer or shorter, of the same curve. A body will descend sooner along an arch of a cycloid than along any other curve drawn between the same points. If water runs out through a hole made in the bottom of a parabolic conoid, the surface of the water will descend equal spaces in equal times. A body descends in a resisting medium with a force only equal to the excess of its gravity above that of an equal bulk of the medium. If a body be thrown downwards in a resisting medium with such force as shall make the resistance of the medium equal to the acceleration of gravity, it will afterwards descend with a uniform motion. If a body descends through any number of inclined planes, it will acquire the same velocity at the end of its fall as though it had fallen through a plane equal in height to the whole, and of the same inclination with the last of them.

DESCRIBENT, a line or surface which, by moving parallel to itself, describes a surface or solid.

DETERMINATE PROBLEM, that which has but one, or at least a limited number of answers, as the following problem, which has but one solution, viz. To describe an isosceles triangle upon a given line whose angles at the base shall be double that at the vertex. But the following has two solutions, viz. To find an isosceles triangle whose area and perimeter are given. A determinate problem may be either simple or linear, plain, solid, or sursolid.

DEXTER, an appellation given to whatever belongs to the right side.

DIAGONAL, a right line drawn across a quadrilateral figure from one angle to the other. Thus *a b* is called a diagonal. It is demonstrable that every diagonal divides a parallelogram into two equal parts,—that two diagonals drawn in any parallelogram bisect each other.

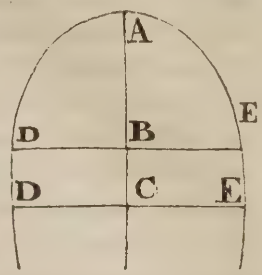


The line *f g*, passing through the middle point of the diagonal of a parallelogram, divides the figure into two equal parts. The diagonal of a square is incommensurable with one of its sides.

That the sum of the squares of the two diagonals of every parallelogram is equal to the sum of the squares of the four sides. This proposition is of great use in the theory of compound motions; for in an oblique angled parallelogram, the greater diagonal being the subtuse of an obtuse and the less of an acute angle which is the complement of the former, if the obtuse angle be conceived to grow until it be infinitely great with regard to the acute one, the great diagonal will be the sum of the two sides, and the less one nothing. Now two contiguous sides of a parallelogram being known, together with the angle they include, it is easy to find one of the diagonals in numbers, and then the foregoing proposition gives the other. This second diagonal is the line which would be described by a body impelled at the same time by two forces which should have the same ratio to each other as the contiguous sides have, and act in those two directions; the body would describe this diagonal in the same time as it would have described either of the contiguous sides in, if only impelled by the force corresponding thereto.

DIAGRAM, the explanation and demonstration of the properties of any figure, whether circle, square, or triangle, &c.

DIAMETER, a right line passing through the centre of a circle, and terminated at each side by the circumference thereof. The properties of the diameter are, that it divides the circumference of a circle into two equal parts; hence we have a method of describing a semicircle upon any line, assuming its middle point for the centre. The diameter of a circle is the greatest of all chords.



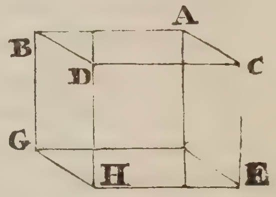
Diameter of a curve is a right line AC, bisecting the right lines DE, DE, drawn parallel to one another, and are either of finite or infinite lengths. Although a right line, bisecting all parallel lines drawn from one point of a curve to another, is taken in a strict sense only for the diameter of a curved line, yet it may not be amiss more generally to define a diameter, in saying that it is that line, whether right or curved, which bisects all parallels drawn from one point of a curve to another.

Diameter conjugate. See Conjugate.

Diameter conjugate of a sphere is the diameter of the semicircle by whose rotation the sphere is generated, in which sense it stands the same as axis.

Diameter of gravity, in any surface or solid, is that line in which the centre of gravity is found to be placed.

Dimension is either length, breadth, or thickness: hence a line has one dimension, viz. length; a superficies two, viz. length and breadth; and a body or solid three, length, breadth, and thickness.



Directrix or **Dirigent** signifies the line of motion along which the describent line or surface is carried in the formation of any plane or solid figure. Thus, if the line AB move along the line AC so that the point A always keeps in the line AC, a parallelogram, as ABCD, will be produced, of which the side AB is the describent, and the line AC the dirigent; and also, if the surface ABCD is supposed to be carried along CE in a position always parallel to itself in its first situation, the solid ADFH will be formed, where the surface AD is the describent, and the line CE the dirigent.

Distances, accessible, are such as can be readily measured.

Distances, inaccessible, are such as cannot be measured by the chain, &c., in consequence of rivers or other obstructions, and are measured by angles from stations.

Divergent, or diverging lines, are those which constantly recede from each other.

Dodecagon, a regular polygon, consisting of twelve equal sides and angles.

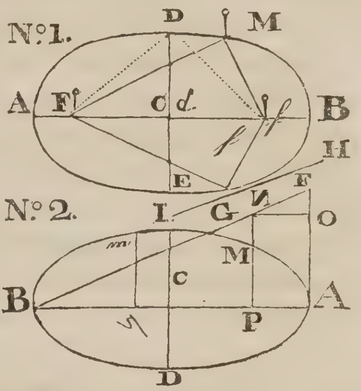
Dodecahedron, a solid contained under twelve equal and regular pentagons. Its solidity is found by multiplying the area of one of the pentagons by twelve, and then this product by one third of the distance of the face from the centre of the dodecahedron, which is the same with the centre of the circumscribing sphere. The side of a dodecahedron, inscribed in a sphere, is the greater part of the side of a cube, inscribed in the same sphere, cut into extreme and mean proportion.

If the diameter of the sphere be 1,0000, the side of the dodecahedron inscribed on it will be ,35682 nearly. All dodecahedrons are similar, and are to one another as the cubes of their sides; their surfaces are also similar, and therefore they are as the squares of their sides. Whence, as ,509282 is to 10,51462 so is the square of the side of any dodecahedron to the superficies thereof; and as ,3637 is to 2,78516, so is the cube of the side of any dodecahedron to its solidity.

Duplication signifies the doubling of any thing, or multiplying by two. The duplication of a cube consists in finding the side of a cube that shall be double in solidity to a given cube.

Eccentric, a term applied to circles and spheres which have not the same centre, and consequently are not parallel, in opposition to concentric, where they are parallel, having one common centre.

Ellipsis, a curved line returning into itself, and produced from the section of a cone, by a plane cutting both its sides, but not parallel to the base. An easy way of describing this curve, when the transverse and conjugate axes AB, ED, are given, is this :—Take the points F *f* in the transverse axis AB, so that the distances CF, C*f*, from the centre C, be equal; or that the lines FD, *f* D, be equal to AC. Then, having fixed two pins in the points F, *f*, which are called the foci of the ellipsis, take a thread equal in length to the transverse axis AB, and fastening the two ends, one to the pin F and the other to *f*, with another pin M stretch the thread tight: then, if the pin M be moved round until it returns to the place from whence it first set out, keeping the thread always extended so as to form the triangle FM*f*, it will describe an ellipsis whose axes are AB, DE. The greater axis, AB, passing through the two foci F, *f*, is called the transverse axis; and the lesser one, DF, the conjugate or second axis: these two always bisect each other at right angles, and the centre of the ellipsis is the point C, where they intersect. Any right line passing through the centre, and terminating by the curve of the ellipsis on each side, is called a diameter; and two diameters which mutually bisect all the parallels to each other bounded by the ellipsis are called conjugate diameters. Any right line, not passing through the centre, but terminated by the ellipsis, and bisected by a diameter, is called the ordinate, or ordinate applicate to that diameter. And a third proportional to two conjugate diameters is called the parameter of that diameter, which is the first of the three proportionals.



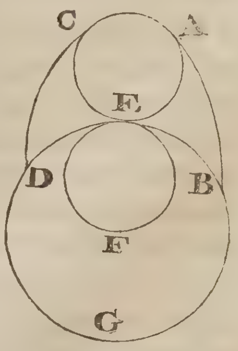
The reason of the name is this: let BA, ED, be any two conjugate diameters of an ellipsis (as No. 2, where there are two axes); at the end A, of the diameter AB, raise the perpendicular AF, equal to the parameter, being a third proportional to AB, ED, and draw the right line BF: then if any point P be taken in BA, and an ordinate PM be drawn, cutting BF in N, the rectangle under the absciss AP and the line PN will be equal to the square of the ordinate PN. For, in drawing NO parallel to AB, it appears that this rectangle, or the square of the ordinate, is less than that under the absciss AP and the parameter AF, by the rectangle under AP and OF or NO and OF, on account of which deficiency the name of an ellipsis was given to this curve.

In every ellipsis, as AEBD, No. 2, the squares of the semiordinates MP, mp, are as the rectangles under the sequents of the transverse axis AP×PB, $Ap \times pB$, made by these ordinates respectively, which holds equally true with the circle, where the squares of the ordinates are equal to such rectangles, as being mean proportionals between the sequents of the diameter. In the same manner, the ordinates to any diameter whatever are as the rectangles under the sequents of that diameter. As to the other principal properties of the ellipsis, they may be reduced to the following propositions:—If from any point M in an ellipsis two right lines, MF, M*f*, No. 1, be drawn to the foci F, *f*, the sum of these two lines will be equal to the transverse axis AB: this is evident from the manner of describing an ellipsis. The square of half the lesser axis is equal to the rectangle, under the sequents of the greater axis, contained between the foci and its vertices. Every diameter is bisected in the centre C. The transverse axis is the greatest, and the conjugate axis the least of all diameters. Two diameters, one of which is parallel to the tangent is the least of all diameters. Two diameters, one of which is parallel to the tangent in the vertex of the other, are conjugate diameters; and, on the contrary, a right line drawn through the vertex of any diameter parallel to its conjugate diameter touches the ellipsis in that vertex. If four tangents be drawn through the vertices of two conjugate diameters, the parallelogram contained under them will be equal to the parallelogram contained under tangents drawn through the vertices of any other two conjugate diameters. If a right line, touching an ellipsis, meet two conjugate diameters produced, the rectangle under the segments of the tangent, between the point of contact and the diameters, will be equal to the square of the same diameter, which is conjugate to that passing through the point of contact. In every ellipsis the sum of the squares of any two conjugate diameters is equal to the sum of the squares of the two axes. In every ellipsis the angles FGI, *f* GH, No. 1, made by the tangent HI, and the lines FG, *f* G, drawn from the foci to the point of contact, are equal to each other. The area of an ellipsis is to the area of the circumscribed circle as the lesser axis is to the greater, and on the contrary, with respect to an inscribed circle, so that it is a mean proportional between two circles, having the transverse and conjugate axes for their diameters. This holds equally true of all the corresponding parts belonging to an ellipsis.

Elliptic. The elliptical space is the area contained within the curve of the ellipsis, which is to that of a circle described on the transverse axis as the conjugate diameter is to the transverse axis; or it is a mean proportional between two circles described on the conjugate and transverse axes.

Enneagon, a polygon with nine sides.

Epicycloid, a curve generated by the revolution of the periphery of a circle, ACE, along the convex or concave side of the periphery of another circle, DGB. The length of any part of the curve that any given point in the revolving circle has described, from the time it touched the circle it revolved upon, shall be double the versed sine of half the arch which all the time touched the circle at rest; as the sum of the diameters of the circles to the semidiameter of the resting circle, if the revolving circle moves upon the convex side of the resting circle; but, if upon the concave side, as the difference of the diameters to the semidiameter of the resting circle. The area of any cycloid or epicycloid is to the area of the generating circle as the sum of double the velocity of the centre, and circular motion to the velocity of the circular motion: and in the same proportion are the areas of segments of those curves to those of analogous segments of the generating circle.



Equal, a term of relation between two or more things of the same magnitude, quantity, or quality.

Equality, that agreement between two or more things whereby they are denominated equal. The equality of two quantities is denoted by two parallel lines placed between them: thus $4+2=6$, that is, 4 added to 2 are equal to 6.

Equation of a curve, an equation expressing the nature of a curve, the relation between an absciss and a corresponding ordinate.

Equiangular, a term given to those figures whose angles are equal, such as the square, equilateral triangle, &c.

Equicrural, the same as isosceles.

Equidistant, an appellation given to things placed at an equal distance from some fixed point or place to which they are referred.

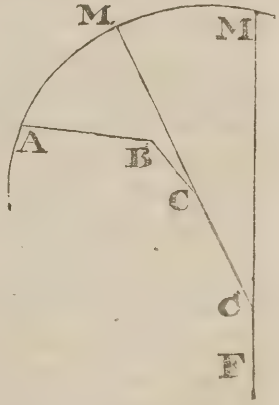
Equilateral, in general, something that has equal sides, as an equilateral angle.

Equilateral hyperbola, one whose transverse diameter is equal to its parameter, and so all the other diameters equal to their parameters. In such a hyperbola, the assympotes always cut each other at right angles in the centre.

Erection, the art of raising or elevating, as the erection of a perpendicular.

Essential, something necessarily belonging to the essence or nature of a thing, from which it cannot be conceived distinct: thus the primary qualities of bodies, as extension, figure, number, &c., are essential or inseparable from them in all their changes and alterations.

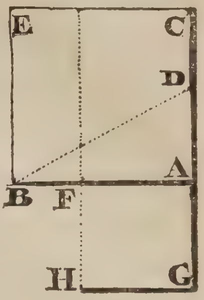
Evolute, a curve which, by being gradually opened, describes another curve. Such is the curve BCF; if a thread, FCM, be wrapped about or applied to the curve and then unwound again, the point M thereof will describe another curve, AMM. The part of the thread MC is called the radius of the evolute, or of the osculatory circle, described on the centre C with the radius MC. When the point B falls in A, the radius of the evolute MC is equal to the arch BC; but, if not, to AB and the arch BC. The radius of the evolute CM is perpendicular to the curve AM, because the radius MC continually touches it; it is evident from its generation that it may be described through innumerable points, if the tangents in the parts of the evolute are produced, until they become equal to their corresponding arches.



External, or Exterior, a term of relation applied to the surface or outside of a body; or that part which appears or presents itself to the eye, touch, &c., in contradistinction to internal.

External angles, the angles on the outside of any right-lined figure, when all the sides are severally produced and taken together, are equal to four right angles.

Extreme and mean proportion is when a line AB is so divided in F that the rectangle under the whole line AB, and the lesser segment FB, are equal to the square of the greater segment AF. Let a square be formed upon the line AB, and one of its sides AC be equally divided in the point D; draw DB and take the line DG, equal to the line BD;—then the square AGHF will be equal to the rectangle FE. As the line AC is equally divided in the point D and lengthened by the line AG, the rectangle CH, together with the square of the line AD, will be equal to the square of the line DG or DB. But the square AE, with the square of the line AD, is also equal to the square of the line DB. Therefore the square AE is equal to the rectangle CH. Taking away from both the rectangle CF, the rectangle FE will be equal to the square FG.



Figure, in conic sections, is the rectangle made under the latus rectum and transversum of the ellipsis and hyperbola.

Figure, of the diameter, is the rectangle under any diameter, and its proper parameter is in the ellipsis and hyperbola, called the figure of that diameter.

Figure, of the superficies, included between one or more lines, and is denominated either rectilinear, curvilinear, or mixed, according as their extremities are bounded by right lines, curved lines, or both.

Finite, something bounded or limited, in contradistinction to infinite.

Focus, in conic sections, is applied to certain points in the parabola, ellipsis, and hyperbola, where the rays reflected from all the parts of these curves concur and meet.

Foci of an ellipsis are two points in the longest axis, on which, as centres, the figure is described. If from the foci two right lines are drawn, meeting each other in the periphery of the ellipsis, their sum will be always equal to the longest axis; and therefore when an ellipsis and its two axes are given, and the foci are required, take half the longest axis in the compasses, and, setting one foot in the end of the shorter, the other will cut the longer in the focus sought.

Focus of a hyperbola is that point in the axis through which the latus rectum passes, from whence if any two right lines are drawn, meeting in either of the opposite hyperbolas, their difference will be equal to the principal axis.

Focus of a parabola, a point in the axis within the figure, distant from the vertex one-fourth part of the latus rectum.

Foot square is the same measure in breadth and length, containing 144 square or superficial inches; the solid foot, or foot cube, is the same in all its three dimensions, length, breadth, and depth, and contains 1728 solid inches.

Frustum, a part of some solid body separated from the rest. The frustum of a cone is the part that remains when the top is cut off by a plane parallel to the base, and is otherwise called a truncated cone.

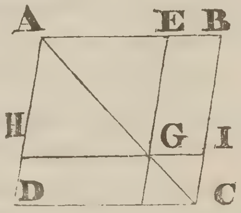
The frustum of a pyramid is also that part which remains after the top is cut off by a plane parallel to its base. The frustum of a globe, or sphere, is any part cut off by a plane the solid content of which may be found in this manner:—To three times the square of the semidiameter of the base add the square of the height; multiply that sum by the height; and the product multiplied by ,5236 will give the solidity of the frustum.

A frustum or portion of any solid, generated by the revolution of any conic section upon its axis, and terminated by any two parallel planes, may be thus compared to a cylinder of the same altitude, whose base is equal to the middle section of the frustum made by a parallel plane. The difference between such frustum and cylinder is always the same in different parts of similar solids, when the inclination of the planes to the axis and the altitude of the frustum are given. In the parabolic conoid this difference vanishes, the frustum being always equal to a cylinder of the same height, upon the section of the conoid that bisects the altitude of the frustum, and is parallel to its bases. In the sphere, the frustum is always less than the cylinder by one-fourth part of a right angled cone of the same height as the frustum, or by one-half of a sphere of a diameter equal to that height; and this difference is always the same in all spheres whatever, when the altitude of the frustum is given. In the cone the frustum always exceeds the cylinder by one-fourth part of the content of a similar cone that has the same height.

GENERATING LINE or FIGURE is that which, by its motion, produces any other plane or solid figure. A right line moved any way parallel to itself generates a parallelogram. Round a point in the same plane, with one end fastened in that point, it generates a circle. One entire revolution of a circle, in the same plane, generates the cycloid; and the revolution of a semicircle round its diameter generates a sphere.

GLOBE, the same as sphere.

GNOMON. If in a parallelogram ABCD the diameter AC be drawn, also two lines EF, HI, parallel to the sides of the parallelogram, and cutting the diameter in the same point G, so that the parallelogram is by these parallels divided into four parallelograms, then are the two parallelograms DG,BG, through which the diameter does not pass, called complements, those through which the diameters pass, EH,FI, are called the parallelograms about the diameter; and a gnomon consists of the two complements and either of the parallelograms about the diameter, viz. GD+HE+EI, or GD+FI+GB.



HEIGHT is a perpendicular from the vertex or top of any right lined figure, upon the base or side subtending it. It is likewise the perpendicular height of any object above the horizon.

HELIX, the spiral.

HEMISPHERE is the half of a globe or sphere, when it is supposed to be cut through its centre in the plane of one of its great circles. The centre of gravity of a hemisphere is five eighths of the radius distant from the vertex.

HEMISPHEROIDAL, an appellation given to whatever approaches to the figure of a hemisphere, but is not exactly so.

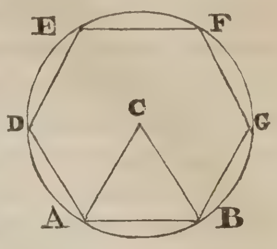
HEPTAGON, a figure consisting of seven sides and as many angles.

HEPTANGULAR, an appellation given to figures which have seven angles.

HETEROGENEOUS, or HETEROGENEAL, that which consists of parts of dissimilar kinds, in opposition to homogeneous.

Heterogeneous, such bodies whose density is unequal in different parts of their bulk; or they are such whose gravities in different parts are not proportionable to the bulks thereof: whereas, bodies equally dense or solid in every part, or whose gravity is proportionable to their bulk, are said to be homogeneous.

HEXAGON, a figure of six sides and angles; if these sides and angles be equal, it is called a regular hexagon. The side of every regular hexagon, inscribed in a circle, is equal in length to the radius of that circle; and, by laying off the radius six times upon the circumference, it will inscribe the hexagon in a circle. To describe a regular hexagon on a given right line AB draw an equilateral triangle ACB, and the vertex C will be the centre of the circle, which will circumscribe the hexagon required, ABDEFG. As 1 is to 1,672, so is the square of the side of any regular hexagon to the area thereof nearly.



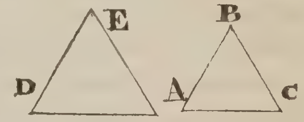
HEXAHEDRON, one of the five Platonic bodies, or regular solids, being the same as the cube.

HOMOCENTRIC, the same as Concentric.

HOMOGENEOUS, or **HOMOGENEAL**, an appellation given to things, the parts of which are similar, or of the same nature and properties.

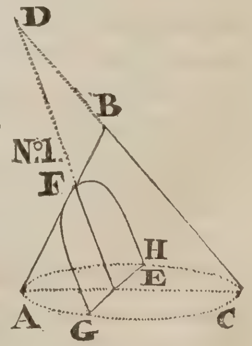
HOMOLOGOUS, an appellation given to the corresponding sides and angles of similar figures, as being proportional to each other.

Thus, in two similar triangles, ABC, DEF, the sides AB and DE, BC and EF, and AC and DF are homologous. And these triangles are to each other as the squares of their homologous sides.

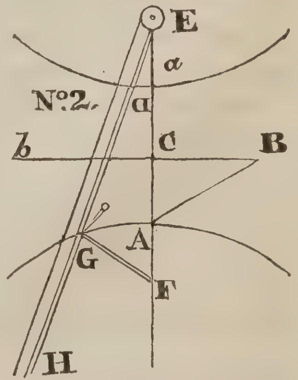


HORIZONTAL, relating to the horizon, or that is taken in, or on a level with the horizon: thus we say a horizontal plane, being that which is perfectly parallel to the horizon, and in no manner inclined thereto.

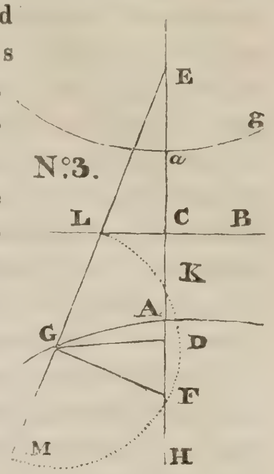
HYPERBOLA, the section GEH of a cone ABC, No. 1, made by a plane, so that the axis EF of the section inclines to the opposite leg of the cone BC, which in the parabola is parallel to it, and in the ellipsis intersects it. The axis of the hyperbolical section will also meet with the opposite side of the cone, when produced above the vertex at D.—No. 1.

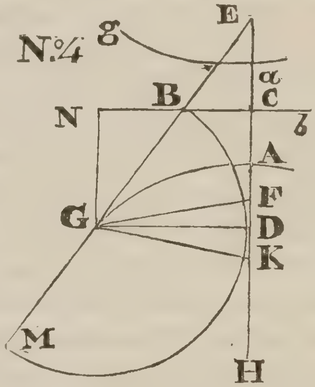


Definitions, No. 1. If at the point E, No. 2, in any plane, the end of the rule EH is so fixed that it may be freely carried round as about a centre, and at the other end of the rule H there is fixed the end of a thread shorter than the rule, let the other end of the thread be placed at the point F in the same plane (but the distance of the points E F must be greater than the excess of the rule above the length of the thread), then let the thread be applied to the side of the rule EH by the pin G and be stretched along it, afterwards let the rule be carried round, and in the mean time let the thread, kept stretched by the pin, be constantly applied to the rule—a line will be described by the motion of the pin which is called the hyperbola. But if the extremity of the same rule which was fixed in the point E is fixed in the point F, and the end of the thread fixed in the point E, and the same performed as before, there will be described another line opposite to the former, which is likewise called a hyperbola, and both together called opposite hyperbolas. These lines may be extended to any distance from the points EF, viz., if a thread be taken of a length greater than that distance. 2. The points E and F are called the foci. 3. The point C, which



bisects the right line between the two focuses, is called the centre of the hyperbola, or of the opposite hyperbolas. 4. Any right line passing through the centre, and meeting the hyperbolas, is called a transverse diameter, and the points in which it meets them their vertices; but the right line, which passes through the centre and bisects any right line terminated by the opposite hyperbolas, and not passing through the centre, is called a right diameter. 5. The diameter which passes through the foci is called the transverse axis. 6. If from A or *a*, the extremities of the transverse axis, there is put a right line AD equal to the distance of the centre C from either focus, and with A as a centre, and the distance AD, there is a circle described meeting the right line which is drawn through the centre of the hyperbola at right angles to the transverse axis in B *b*, the line B *b* is called the second axis. 7. Two diameters, either of which bisects all the right lines parallel to the other, and which are terminated both ways by the hyperbola or opposite hyperbolas, are called conjugate diameters. 8. Any right line not passing through the centre, but terminated both ways by the hyperbola or opposite hyperbolas, and bisected by a diameter is called an ordinate applied, or simply an ordinate to that diameter: the diameter likewise, which is parallel to the other right line ordinately applied to the other diameter, is said to be ordinately applied to it. 9. The right line which meets the hyperbola in one point only, but produced both ways falls without the opposite hyperbolas, is said to touch it at that point, or is a tangent to it. 10. If, through the vertex of the transverse axis, a right line be drawn equal and parallel to the second axis, and is bisected by the transverse axis, the right lines drawn through the centre and the extremities of the parallel line are called assymptotes. 11. The right line drawn through the centre of the hyperbola, parallel to the tangent, and equal to the sequent of the tangent between the assymptotes, and which is bisected in the centre, is called the second diameter of that which is drawn through the point of contact. 12. A third proportional to the two diameters, one of which is the transverse, the other second to it is called the right line or parameter of that diameter, which is the first of the three proportionals. And, 13. lastly, No. 5. If upon two right lines A *a*, B *b*, mutually bisecting each other at right angles, the opposite hyperbolas AG, *ag* are described, and if upon the same right lines there are described two other opposite hyperbolas BK, *bk*, of which the transverse axis B *b* is the second axis of the two first, and the second axis of the two last, A *a*, is the transverse axis of the two first—these four are called conjugate hyperbolas, and their assymptotes shall be common. Proposition I. No. 2. The square of half the second axis is equal to the rectangle contained by the right lines between the foci and the vertex of the transverse axis. Let A *a* be the transverse


N°.3.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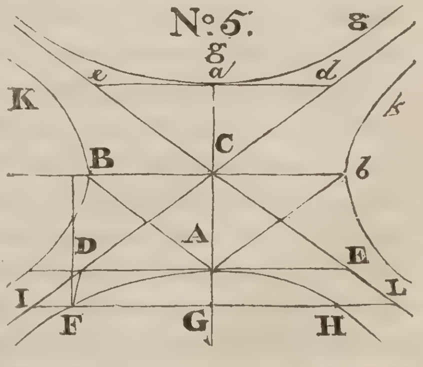
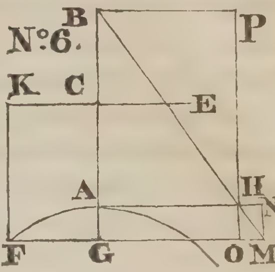
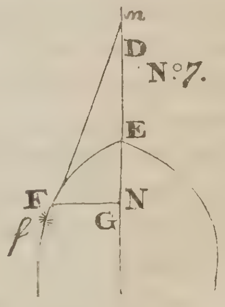

N°.5.

axis, C the centre, E and F the foci, and B *b* the second axis, which is evidently bisected in the centre C from this definition; let AB be joined: then, as by definition 6, AB and CF are equal, the squares of AC and CB together will be equal to the square CF, that is, to the square of AC and the rectangle AF *a* together; therefore, taking away the square of AC, which is common, the square of CB will be equal to the rectangle AF *a*. Proposition II. If from any point G, Nos. 3 and 4, of the hyperbola, a right line GD be drawn at right angles to the transverse axis A *a*, and if from the same point there is drawn the right line GF to the focus nearest to that point, the half of the transverse axis CA will be to the distance of the focus from the centre, viz. CF, as the distance of the perpendicular CD is to the sum of half of the transverse axis, and the right line drawn to the focus. Let GE be drawn to the other focus, and, on the axis A *a* produced, let there be set off AH equal to GF; then with the centre G and the distance GF describe a circle cutting the axis *a* A in K and F, and the right line EG in the points L and M: then, as EF is double CF, and FK double FD, EK shall also be double CD, and as EL or A *a* is double CA, and LM double GF or AH, EM shall also be double CH: but because the circle EL or A *a*: EF: : EK: EM: and taking their halves, it will be as CA: CF: : CD: CH. Proposition III. The same being proposed, if from A, the extremity of the transverse axis nearest to the point G, there is set off a right line AH on the axis produced equal to the distance of the point G from the focus F nearest to the said extremity, the square of the perpendicular GD shall be equal to the excess of the rectangle EHF, contained under the segments between H (the extremity of the right angle AH) and the foci, above the rectangle AD *a*, contained under the segments cut off between the perpendicular and the extremities of the axis. For the right line CH is in any manner cut in A, the squares of CA and CH together will be equal to twice the rectangle ACH, and the square of AH (7. 2.), that is, because CA, CF, CD, CH, are proportionals to twice the rectangle FCD and to the square of AH or GF, that is, to twice the rectangle FCD and the squares of FD and DG, that is, to the squares of FC, CD, and DG (7. 2): therefore the two squares of CA and CH are equal to the three squares of FC, CD, and DG; and, taking away the squares of CA and CF from both sides, the remaining rectangle EHF will be equal to the remaining rectangle AD *a* and to the square of DG (6. 2). Proposition IV. If from any point G of the hyperbola there be drawn a right line parallel to the second axis B *b* meeting the transverse axis A *a* in D, the square of the transverse axis shall be to the square of the second axis as the rectangle contained under the segments of the transverse axis between the parallels and its extremes to the square of the parallel. Proposition V. No. 4. If from any point G of the hyperbola there is drawn a right line parallel to the transverse axis A *a* meeting the second axis in N, the square of the second axis shall be to the square of the transverse as the sum of the squares of half of the second axis and its segment between the centre and the right line to the square of the line itself. Proposition VI. It is another property of the hyperbola that the assymptotes, D *d*, E *e*, never absolutely meet with the curve. Proposition VII. If through any point F, No. 5, of the hyperbola, there is drawn a line IFL parallel to the


N°6.


N°7.

second axis and meeting the assymptotes in I and L, the rectangle contained under the right lines which are intercepted between the assymptotes and the hyperbola are equal to the square of half the second axis. Proposition VIII. If from any point, F, of the hyperbola there is drawn to the transverse diameter, AB, a right line ordinately applied to it, FG, and from the extremity of the diameter there is drawn AH perpendicular to it, and equal to the latus rectum, the square of the ordinate shall be equal to the rectangle applied to the latus rectum, being of the breadth of the abscissa between the ordinate and the vertex, and which exceeds it by a figure alike situated to that which is contained under the diameter and the latus rectum. Join BH, and from the point G let there be drawn GM parallel to AH and meeting BH in M, and through M let there be drawn MN parallel to AB, meeting AH in N, and let the rectangles MNHO, BAHP, be completed. For the rectangle AGB is to the square of GF as AB is to AH and as GB is to GM, the rectangle AGB is to the rectangle AGM, AGB shall be to the square of GF as the same AGB to the rectangle AGM: therefore the square of GF is equal to the rectangle AGM, which is applied to the latus rectum AH, having the breadth AG, and exceeding the rectangle HAGO by the rectangle MNHO, as BAHP, from which excess the name of hyperbola was given to this curve.

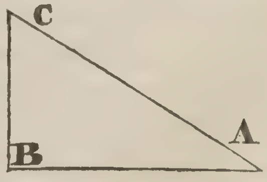
An easy method to describe the hyperbola, having the transverse diameter DE, and the foci N *n*, given.—From N at any distance, as NF, strike an arch; and with the same opening of the compasses, with one foot in E the vertex, set off EG equal to NF in the axis continued; then with the distance GD, and one foot in *n*, the other focus, cross the former arch in F. So F becomes a point in the hyperbola: and by this method repeated may be found any other point *f* further on, and as many more.

Ambigenal hyperbola is that which has one of its infinite legs inscribed and the other circumscribed. Equilateral hyperbola is that wherein the conjugate axes are equal.

HYPERBOLIC CYLINDROID is a solid figure, viz. two opposite hyperbolas being joined by the transverse axis, and through the centre a right line drawn at right angles to the axis, and about that as an axis the hyperbolas being supposed to revolve, by such revolution a body will be generated which is called the hyperbolic cylindroid, whose bases and all sections parallel to them will be circles.

Hyperbolic line is sometimes used for the hyperbola itself. In this sense, the plane surface terminated by the curved line is called the hyperbola, or hyperbolic space, and the curved line that terminates it the hyperbolic line.

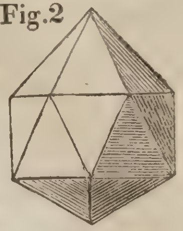
HYPOTHENUSE, the longest side AC of a right-angled triangle ABC, or it is that side which subtends the right angle B.



HYPOTHESIS denotes something supposed to be true, or taken for granted, in order to prove or illustrate a point in question.

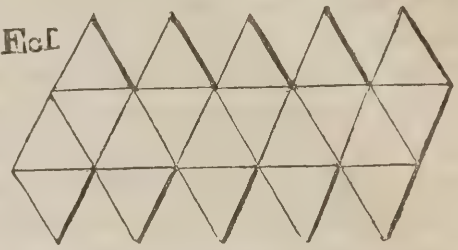
ICOSAHEDRON, a regular solid, consisting of twenty triangular pyramids, whose vertexes meet in the centre of a sphere, supposed to circumscribe it, and have their height and bases equal: therefore the solidity of one of those pyramids multiplied by twenty, the number of bases, gives the solid content of the icosahedron.

Fig.2



If figure 1 be drawn on pasteboard, and cut about half way through, and afterwards folded together, it will represent an icosahedron, as fig. 2.

FIG. I



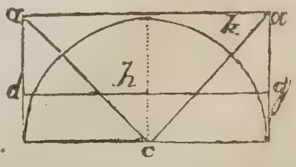
INCLINATION signifies the approach, tendency, or leaning of two lines or two planes towards each other, so as to make an angle. Inclination of a right line to a plane is the acute angle which that line makes with another right line drawn in the plane, through the point where the inclined line intersects it, and through the point where it is also cut by a perpendicular drawn from any point of the inclined plane.

INCLINED PLANE, that which makes an oblique angle with the horizon.

INCOMMENSURABLE is used where two lines, when compared to each other, have no common measure, however small, that will exactly measure them both. And, in general, two quantities are said to be incommensurable when no third quantity can be found that is an aliquot part of both. Such are the diagonal and side of a square; although each of these lines have infinite aliquot parts, as the half, third, &c., yet not any part of the one, let it be ever so little, can possibly measure the other. As two surfaces, which cannot be measured by a common surface, are said to be incommensurable in power.

INDEFINITE, that which has no certain boundary.

INDIVISIBLES, the elements or principles into which any body or figure may be ultimately resolved, which elements are supposed to be infinitely small: a line may be said to consist of points, a surface of parallel lines, and a solid of parallel and similar surfaces, because each of these elements is supposed indivisible; if in any figure a line be drawn through the elements perpendicularly, the number of points in that line will be the same as the number of the elements; from whence we may see that a parallelogram, prism, or cylinder, is resolvable into elements, or indivisibles, and all equal to each other, parallel and like to the base; a triangle into lines parallel to the base, but decreasing in arithmetical proportion, and so are the circles which constitute the parabolic conoid, and the plane of a circle or surface of an isosceles cone. The method of indivisibles is found of great use sometimes in shortening mathematical demonstrations; for instance, suppose a cylinder, a hemisphere, and an inverted cone to have the same base and altitude, and cut by infinite planes parallel to the base, of which *dg* is one. It is plain the square of *dh* will be every where equal to the square of *kc* (the radius of the sphere), the square *hc*=*eh* square; and consequently such



circles are to one another as the squares of the radii; all the circles of the hemisphere will be equal to all those of the cylinder, deducting all those of the cone; therefore the cylinder, deducting the cone, is equal to the hemisphere: but it is understood that the cone is one-third of the cylinder, consequently the sphere must be two-thirds of it.

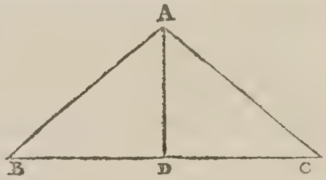
Infinite, or infinitely great line, denotes an indefinite or indeterminate line, to which no certain bounds or limits are prescribed.

Inscribed. A figure is said to be inscribed in another when all its angles touch the sides or planes of the other figure.

Internal signifies whatever is within another.

Isoperimetrical figures are such as have equal perimeters or circumferences. Of isoperimetrical figures, that is the greatest which contains the greatest number of sides, or the most angles; consequently the circle is the greatest of all figures that have the same perimeter it has. Of two isoperimetrical triangles having the same base, whereof two sides of one are equal and of the other unequal, that is the greater whose two sides are equal. Of those figures whose sides are equal in number, that is the greatest which is equilateral and equiangular.

Isosceles triangle, one that has two equal sides, as AB, AC, where the side AB is equal to AC. In every isosceles triangle the angles ABC, ACB, subtended by the equal sides, are equal; and a line, AD, bisecting the base BC in D, is perpendicular to it.

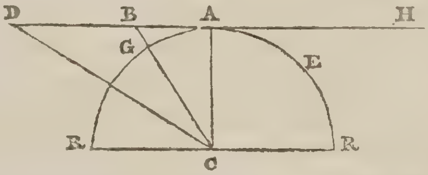


Latus rectum, in conic sections, a constant line, as parameter.

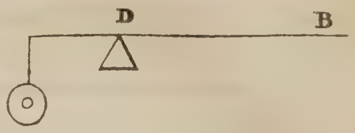
Latus transversum, in the hyperbola, that part of the tranverse diameter intercepted between the vertices of the two opposite sections.

Latus primarium, a right line belonging to a conic section drawn through the vertex of the section and within it.

Levelling, the art of finding a line parallel to the horizon at one or more stations, in order to determine the height of one place with regard to another. A truly level surface is a segment of a spherical surface, which is concentric to the globe of the earth. A true line of level is an arch of a great circle, which is imagined to be described upon a truly level surface. The apparent level is a straight line drawn tangent to an arch, or line of true level. Every point of the apparent level, except the point of contact, is higher than the true level: thus, let EAG be an arch of a great circle drawn upon the earth; to a person who stands upon the earth at A, the line HD is the apparent level parallel to his rational horizon RR; but this line, the further it is extended from its station A, the further it recedes from the centre; for BC is longer than AC, and DC is longer than BC, &c.

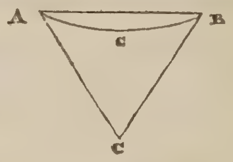


Lever, an inflexible right line, rod, or beam, supported in a single point on a fulcrum, used for raising weights, being either void of weight itself, or at least having such weight as may be counterbalanced. In a lever there are three things to be considered: the weight to be sustained, as O, the power by which it is to be raised, as B, and the fulcrum D, whereon the lever is sustained. In a lever, the moving force or power that has to the weight the same ratio which the distance of the line of direction of the weight from the fulcrum has to the distance of the direction of the power from the fulcrum sustains the weight; and therefore, if it be but little increased, it will raise the weight. It is manifest from this theorem that the spaces which are described by a power and weight, according or contrary to their propensions, are proportionable to the distances of the lines of direction from the fulcrum; but the velocities are proportionable to these spaces, and consequently will also be proportionable to the distance.

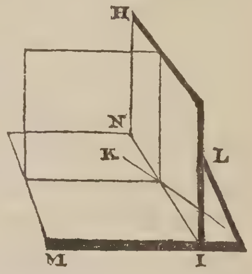


LIMITED PROBLEM, a problem that admits of but one solution: for instance, to make a circle pass through three given points not lying in a right line.

LINE, a quantity extended in length only, without any breadth or thickness. There are two kinds of lines, viz., right lines and curved lines. If the point A move towards B, by its motion it describes a line which, if the point go the nearest way towards B, will be a right or straight line, whose distinction therefore is the nearest or shortest distance between any two points, or a line all whose points tend the same way. If the points go about as one of the lines ABC, or A *c* B, it will trace out a crooked line, as in the upper, A *c* B, or else two more straight ones, as in the lower, ACB. Right lines are all of the same, but curves are of an infinite number of species.



Lines in perspective are, first, the geometrical line, which is a right line drawn in any manner on the geometrical plane; secondly, the terrestrial or fundamental line, being a right line wherein the geometrical plane and that of the draught intersect each other; such is the line NI, formed by the intersection of the geometrical plane LM, and the perspective plane HK; thirdly, the line of the front is any right line parallel to the terrestrial line; fourthly, the vertical line, the common section of the vertical and the draught; fifthly, the visual line, the line or ray imagined to pass from the object to the eye; sixthly, the line of station, the common section of the vertical and geometrical planes; seventhly, the line of an object whence the appearance is sought for in the draught.



Line of direction, that line wherein a body actually moves, or would move were it not prevented. It also denotes the line that passes through the centre of gravity of the heavy body to the centre of the earth, which must also pass through the fulcrum, or support, of the heavy body, without which it would fall.

LINEAR PROBLEM, that which may be solved geometrically by the intersection of two right lines. This is called a simple problem, and is capable of but one solution.

LOZENGE, the rhombus, a quadrilateral figure, consisting of four equal and parallel sides, two of whose opposite angles are acute and the other two obtuse; the distance between the two obtuse ones, being always equal to the length of one side: when the sides are unequal, this figure is called a rhomboides.

Mean, in general, denotes the middle between two extremes: as mean distance, mean proportion, &c.

Mensuration denotes the art of measuring lines, superficies, solids, &c.

Minute, the sixtieth part of a degree of a circle.

Oblique, a deviation from the perpendicular. An oblique angle is either an acute or an obtuse one, or any angle except a right one.

Oblique line, that which falling on another line makes oblique angles with it, viz., one acute, the other obtuse.

Oblong denotes a figure that is longer than broad; such is a parallelogram, an ellipsis, &c.

Obtuse, in opposition to acute, sharp, &c.

Occult lines are lines made scarcely perceptible, or those made by a fine pencil, which can be removed when the work is completed.

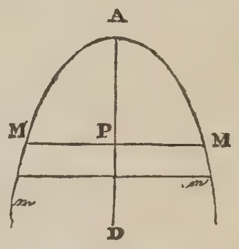
Octagon, a figure of eight sides and angles; and, when all the sides and angles are equal, it is a regular octagon.

Octahedron, one of the five regular bodies, consisting of eight equal and equilateral triangles. The square of the side of the octahedron is to the square of the diameter of the circumscribing sphere as one to two. If the diameter of the sphere be 2, the solidity of the octahedron inscribed in it will be 1.33333 nearly. The octahedron is two pyramids put together at their bases, therefore its solidity may be found by multiplying the quadrangular base of either by one-third of the perpendicular height of one of them, and doubling the product.

Opposite sections are two hyperbolas made by cutting two opposite cones by the same plane.

Opposition, the relation of two things between which a line may be drawn perpendicular to both.

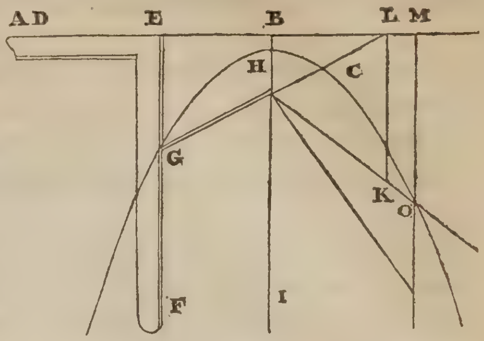
Ordinates, or ordinate applicates, are parallel lines terminating in a curve, and bisected by a diameter, as AD. The half of these, as MP, *mp*, is properly the semi-ordinate, though commonly called ordinate.



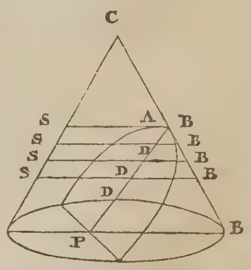
Orthography, the art of drawing or delineating the plan of any object, and of expressing the heights or elevations of each part. It is called orthography from its determining objects by perpendicular lines falling on a geometrical plane.

In perspective, it is the side or plane that lies parallel to a straight line that may be imagined to pass through the outward convex points of the eye, continued to a proper or convenient length.

Oval, an oblong curvilinear figure, otherwise called ellipsis. See Ellipsis.



PARABOLA, a figure arising from the section of a cone when cut by a plane parallel to one of its sides. To describe a parabola, draw a right line AB, and assume a point C, without it; then, in the same plane, with the line and point, place a square rule DEF so that the side DE may be applied to the right line AB, and the other EF turned to the side in which the point C is situate. This done, and the thread FGC, exactly of the length of the side of the rule EF, being fixed at one end to the extremity of the rule F, and the other to the point C, slide the side of the rule DE along the right line AB, and, by the means of a pin G, continually apply the thread to the side of the rule EF, to keep it always stretched as the rule is moved along: the point of the pin will describe a parabola, GHO. The right line, AB, is called the directrix. The point C is the focus of the parabola. All perpendiculars to the directrix, as LK, MO, &c., are called diameters; the points where these cut the parabola are called its vertices; the diameter, BI, which passes through the focus C, is called the axis; and its vertex, H, the principal vertex; a right line terminated on each side by the parabola and bisected by a diameter is called the ordinate applicate, or simply the ordinate to that diameter. A line equal to four times the segment of any diameter, intercepted between the directrix and the vertex where it cuts the parabola, is called the latus rectum, or parameter, of that diameter. A right line which touches the parabola in one point only, and, being produced on each side, falls without it, is a tangent to it in that point. Any right line, as GE, drawn from any point of the parabola, G, perpendicular to AB, is equal to a line GC drawn from the same point to the focus. This is evident from the description, for the length of the thread, FGC, being equal to the side of the rule, EF, if the part, FG, common to both, be taken away, there remains EG=GC. The reverse of this proposition is equally evident, viz., if the distance of any point from the focus of a parabola be equal to the perpendicular drawn from it to the directrix, that point will fall in the curve of the parabola.



PARABOLIC CONOID, a solid generated by the rotation of a parabola about its axis: its solidity is equal to half of its circumscribing cylinder. The circles conceived to be the elements of this figure are in arithmetical proportion, decreasing towards the vertex. A parabolic conoid is to a cylinder of the same base and height as one to two, and to a cone of the same base and height as one and a half to one.

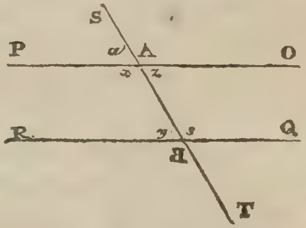
Parabolic cuneus, a solid figure formed by multiplying all the DB's into the DS's. Or, which amounts to the same, on the base APB erect a prism whose altitude is AS; this will be a parabolic cuneus, which will be equal to the parabolic pyramidoid, as the compound rectangles in one are severally equal to all the component squares in the other.

Parabolic pyramidoid, a solid figure generated by supposing all the squares of the ordinate applicates in the parabola so placed that the axis passes through all the centres at right angles, in which case the aggregate of the planes will form the parabolic pyramidoid. The solidity thereof is obtained by multiplying the base by half the altitude, the reason of which is quite obvious; for the component planes being series of arithmetical proportionals, beginning from the cypher, their sum will be equal to the extremes multiplied by half the number of terms.

Parabolic space. The segment of a parabolic space is that space included between two ordinates.

Parabolic spindle is eight-fifteenths of its circumscribing cylinder.

Parallel, an appellation given to lines, surfaces, and bodies every where equidistant from each other, and which, though infinitely produced, would never meet: thus, the line OP is parallel to QR. If two parallels, OP and QR, be cut by a transverse line, ST in A and B, the alternate angles, x and y, are equal. The external angle, u, is equal to the internal opposite one, y. The two internal opposite ones, x and s, are also equal to the two right angles.

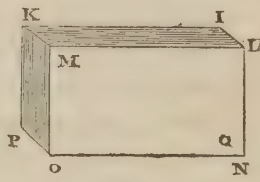


It is shown in the principles of optics that if the eye be placed between two parallel lines they will appear to converge towards a point opposite the eye, and, if they run to such length that the distance between them be but as a point thereto, they will then appear to coincide. Parallel lines are described by letting fall equal perpendiculars, and drawing lines through the extremes.

Parallel planes are such planes as have all their perpendiculars drawn between them equal to each other.

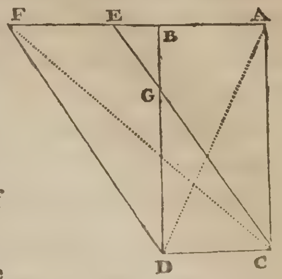
Parallelopiped, a regular solid comprehended under six parallelograms, the opposite being similar, parallel, and equal. All parallelopipeds, prisms, cylinders, &c., whose bases and heights are equal, are themselves equal.

A diagonal plane divides a parallelopiped into two equal prisms, so that a triangular prism is half a parallelopiped upon the same base and of the same altitude. All parallelopipeds, prisms, cylinders, &c., are in a ratio compounded of their bases and altitudes: therefore, if their bases be equal, they are in proportion to their altitudes. All parallelopipeds, cylinders, cones, &c., are in a triplicate ratio of their homologous sides, and also of their altitudes. Equal parallelopipeds, prisms, cones, cylinders, &c., reciprocate their bases and altitudes. To measure the surface and solidity of a parallelopiped, find the areas of the parallelogram, ILMK, LMON, and OMKP; add these into one sum and multiply it by two; the factum will be the surface of the parallelopiped. Or if the base, ILMK, be multiplied by the altitude, MO, the product will be the solidity.



Parallelism, the situation or quality whereby any thing is denominated parallel.

Parallelogram, a quadrilateral right-lined figure, whose opposite sides are parallel and equal to each other. It is generated by an equal motion of a right line always parallel to itself. When it has all its four angles right, and only its opposite sides equal, it is called a rectangle. When the angles are all right, and the sides equal, it is called a square. If all the sides are equal, and the angles unequal, it is called a rhombus or lozenge: and, if the sides and angles be unequal, it is called a rhomboides.

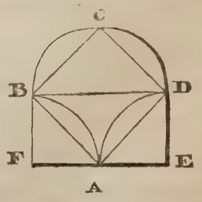


In every parallelogram, as ABCD, a diagonal, DA, divides it into two equal parts; the angles diagonally opposite BC and AD are equal; the opposite angles of the same side CD and AB, &c., are together equal to two right angles, and the two sides together greater than the diagonal. Two parallelograms, ABCD and ECDF, on the same or equal base, CD, and the same height, AC, or between the same parallels, AF,CD, are equal: and the two triangles, CDA and CDF, on the same base, and of the same height, are also equal.

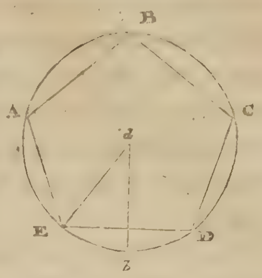
Every triangle, CFD, is half a parallelogram, ACDB, upon the same or an equal base, CD, and of the same altitude, or between the same parallels. A triangle is equal to a parallelogram having the same base and half the altitude, or half the base and the same altitude. Parallelograms are, in a given ratio, compounded of their bases and altitudes. If the altitudes be equal, they are as the bases, and conversely. In similar parallelograms and triangles, the altitudes are proportional to the homologous sides, and the bases are cut proportionably thereby. Similar parallelograms and triangles are in duplicate ratio to their homologous sides, as also of their altitudes and the segments of their bases; they are therefore as the squares of the sides, altitudes, and homologous segments of the bases. In every parallelogram the sum of the squares of the two diagonals is equal to the sum of the squares of the four sides. For, if the parallelogram be rectangular, it follows that the two diagonals are equal; and, consequently, the square of a diagonal, or, which comes to the same, the square of a hypothenuse of a right angle, is equal to the squares of the sides. If the parallelogram be not rectangular, the diagonals will be unequal, which is the general case. The proposition then becomes of vast extent; for instance, in the whole theory of compound motions, &c. To find the area of the rectangled parallelogram, ABCD, find the length of the sides, AB and AC; multiply AC, supposed=345 into AB=33; the product will be the area of the parallelogram, namely, 11,385. Rectangles are in a ratio compounded of their sides AC and AB. If, therefore, three lines be continually proportional, the square of the middle one is equal to the rectangle of the two extremes; and, if four lines be proportional, the rectangle under the two extremes will be equal to that under the two middle terms. Other parallelograms, not rectangular, have their areas found by resolving them by diagonals into two triangles, and adding the areas of the separate triangles into one sum.

PARAMETER, a constant line, otherwise called latus rectum. The parameter is said to be constant, because, in the parabola, the rectangle under it and any absciss is always equal to the square of the corresponding semi-ordinate; in the ellipsis and hyperbola, it is a third proportional to the conjugate and transverse axis.

PELECOIDES, a figure contained under the two inverted quadrantal arcs, AB and AD, and the semicircle BCD. The area of the pelecoides is demonstrated to be equal to the square AC, and that again to the parallelogram EB. It is equal to the square AC, because it wants of the square on the left the two segments AB and AD, which are equal to the two segments BC and CD, by which it exceeds on the right.



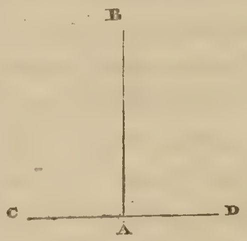
PENTAGON, a figure of five sides and five angles. If the five sides be equal, the angles are also equal; and the figure is called a regular pentagon. Such is ABCDE, inscribed in the circle. The most considerable property of a pentagon is, that one of its sides, DE, is equal in power to the sides of a hexagon and a decagon inscribed in the same circle, ABCDE, that is, the square of the side DE is equal to the sum of the squares of the sides *d* E, and E *b*. The area of the pentagon, like any other polygon, may be obtained by resolving it into triangles. The dodecahedron, which is the fourth regular solid, consists of twelve pentagons.



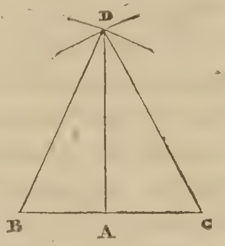
Perimeter, the bounds or limits of any figure or body. The perimeter of surfaces or figures are lines, those of bodies are surfaces. In circular figures, instead of perimeter, we say circumference, or periphery.

Periphery, the circumference of a circle, ellipsis, or any other regular curvilinear figure.

Perpendicular, a line falling directly upon another, so as to make equal angles on each side, called also a normal line: thus, the line AB is perpendicular to the line CD; that is, it makes right angles therewith.

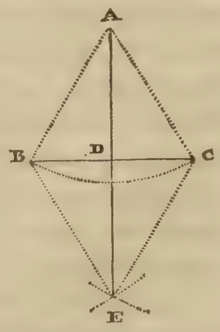


From the nature of a perpendicular it follows, That the perpendicularity is natural, that is, if a line, as AB, be perpendicular to another CD, that is also perpendicular to the first. That only one perpendicular can be drawn from one point in the same place. That if a perpendicular be continued through the line it was drawn perpendicularly to, the continuation will also be perpendicular to the same. That if there be two points of a right line, each of which is of an equal distance from two points of another right line, that line will be perpendicular to the other. That line which is perpendicular to another, is also perpendicular to all the parallels of the other. A perpendicular line is the shortest of all those which can be drawn from the same point to the same right line. The shortest distance of a point from a line is a right line drawn from the point perpendicular to the line or plane, the altitude of a figure is a perpendicular let fall from the vertex to the base.

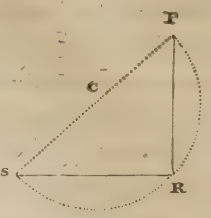


To raise from the point A a line perpendicular to the line BC, make AB=AC, and from the points B and C as centres, with the same opening of the dividers, describe two arches cutting each other in the point D, and the line DA shall be the perpendicular described; that is, the angles, DAB, DAC, shall be equal, because all the sides of the triangle DAB will be equal to all the sides of the triangle DAC.

To let fall a perpendicular upon a line, BC, from a point given without it, A, on the point A describe an arch which shall cut the line in the points B and C: then making the equilateral triangle BEC, the line AE will be perpendicular to the line BC. The triangles, ABD and ACD, have the side AD common, and AB is equal to AC, and the angle BAD is equal to the angle CAD, they are equal in every respect, and the angle D equal on each side: therefore, by this definition, the line AD is perpendicular to BC.



To erect a perpendicular at the end of a given line, suppose at R, open the dividers to any convenient distance, and setting one foot in C, draw the semicircle PRS. Lay a ruler from S through C, and it will find the point P in the circumference, then draw PR, which is the perpendicular required. For the angle PRS, being in a semicircle, must be a right one; consequently, PR must be perpendicular to SR. A line is said to be perpendicular to a plane when it is perpendicular to more than two lines drawn in that plane; and a plane is said to be perpendicular to another plane, when a line in one plane is perpendicular to the other.



Perpendicular to a parabola is a right line cutting the parabola in the point in which any other right line touches it, and is also itself perpendicular to that tangent.

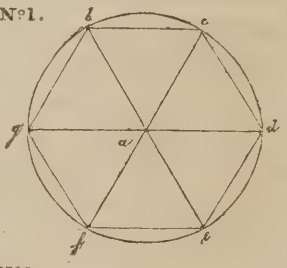
Plane denotes a plain surface, or one that lies evenly between its bounding lines: and as a right line is the shortest extension from one point to another, so a plain surface is the shortest extension from one line to another.

Planimetry, that part of geometry which considers lines and plain figures, without considering their height or depth.

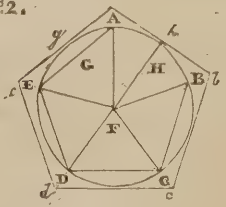
Point is a quantity which has no parts, or which is indivisible. Points are the ends and extremities of lines. If a point was supposed to be moved, it would, by its own motion, describe a line.

Polygon, a figure with many sides, or whose perimeter consists of more than four sides; such are the pentagon, hexagon, heptagon, &c. Every polygon may be divided into as many triangles as it has sides, for if you assume a point, as A, any where within the polygon, and from thence draw lines to every angle *ab*, *ac*, *ad*, &c., they will make as many triangles as the figure hath sides. Then, if the polygon has six sides, the double of that is twelve, from whence take four, and there remain eight, therefore all the angles, *bcdefg*, of that polygon taken together, are equal to eight right angles. For the polygon, having six sides, is divided into six triangles, and the three angles of each are equal to two right ones; so that all the angles together make twelve right ones: but each of these triangles has one angle in the point *a*, and by it they complete the space round the same point; and all the angles about a point are known to be equal to four right ones, therefore the four taken from twelve leave eight, the sum of the right angles of the hexagon. It is therefore plain, the figure has twice as many right angles as it has sides, excepting four. Every polygon circumscribed about a circle is equal to a rectangled triangle, one of whose legs shall be the radius of the circle, and the other the perimeter (or sum of all the sides) of the polygon. Every regular polygon is equal to a rectangled triangle, one of whose legs is the perimeter of the polygon, and the other a perpendicular drawn from the centre to one of the sides of the polygon. And every polygon circumscribed about a circle is larger than it; and every polygon inserted is less than the circle. The perimeter of every polygon circumscribed about a circle is greater than the circumference of the circle, and the perimeter of every polygon inscribed is less. A circle is equal to a right-angled triangle, whose base is the circumference of the circle, and its height the radius of it.

N°1.



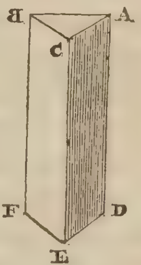
N°2.



For this triangle will be less than any polygon circumscribed, and greater than any inscribed; because the circumference of the circle, which is the base of the triangle, is greater than the compass of any inscribed, therefore it will be equal to the circle. For if the triangle be greater than any thing that is less than the circle, and smaller than any thing that is greater than the circle, it follows that it must be equal to the circle. This is called the quadrature or squaring of the circle; that is, to find a right-lined figure equal to a circle, upon a supposition that the basis given is equal to the circumference of the circle: but actually to find a right line equal to the circumference of a circle is not yet discovered geometrically. On a regular polygon to circumscribe a circle, or to circumscribe a regular polygon upon a circle, bisect two of the angles of the given polygon, A and B (No. 2), by the right lines AF, BF, and on the point F where they meet, with the radius AF, describe a circle which will circumscribe the polygon. Next, to circumscribe a polygon, divide 360 by the number of sides required, to find *e* F *d;* which set off from the centre F, and draw the line *d e*, on which construct the polygon as in the following problem:—On a given line to describe any given regular polygon, find the angle of the polygon in the table, and in E set off an angle equal thereto; then drawing EA=ED through the points EAD, describe a circle, and in this applying the given right line as often as you can, the polygon will be described. To find the sum of all the angles in any given regular polygon, multiply the number of sides by 180°, from the product subtract 360°, and the remainder will be the sum required: thus, in a pentagon, $180 \times 5 = 900$, and $900 - 360 = 540$ = the sum of all the angles in a pentagon. To find the area of a regular polygon, multiply one side of the polygon by half the number of sides; and then multiply this product by a perpendicular let fall from the centre of the circumscribing circle, and the product will be the area required: thus, if AB (the side of a pentagon, $=54 \times 2\frac{1}{2}$,)=135, and 135×29 (the perpendicular)=3915, the area required. To find the area of an irregular polygon, let it be resolved into triangles, and the sum of the areas of these will be the area of the polygon.

POLYHEDRON denotes a body, or solid, comprehended under many sides or planes.

PRISM, an oblong solid contained under more than four planes, whose bases are equal, parallel and alike situated. The prism is generated by the motion of a rectilinear figure, as ABC, descending always parallel to itself along the right line AE. If the describent be a triangle, the body is said to be a triangular prism; if square, a quadrangular one. From the principle of the prism, it is evident it has two equal and opposite bases, ABC and EDF; and it is terminated by as many parallelograms as the base consists of sides; and all the sections of a prism, parallel to its base, are equal. Every triangular prism may be divided into three equal pyramids. To measure the surface of any prism, find the area of each side, whether a triangle, parallelogram, or other rectilinear figure, as directed under these articles, and the sum of all these taken together make the whole superficies of the prism.



The solid content of a given prism may be found thus: let the area of the base of the prism be measured, as directed under the article TRIANGLE; and this area multiplied by the height, and the product will give the solid content of the prism.

PRISMOID, a solid figure, bounded by several planes whose bases are right-angled parallelograms, parallel and alike situate.

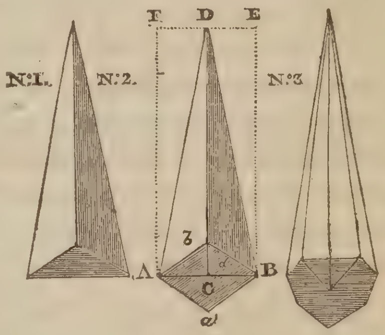
Problem is a proposition, wherein some operation or construction is required, as to divide a line or angle, erect or let fall perpendiculars, &c. A problem consists of three parts; the proposition, which expresses what is to be done; the solution, wherein the several steps whereby the matter required is to be effected, are rehearsed in order; and, lastly, the demonstration, wherein is shown, that by doing the several things prescribed in the solution the object required is obtained.

Product, the factum of two or more numbers, or lines, &c., into one another: thus $5 \times 4 = 20$, the product required.

Proposition is either some truth advanced and shown to be such by demonstration, or some operation proposed and its solution shown. If the proposition be deduced from several theoretical definitions compared together, it is called a theorem; if from a praxis, or series of operations, it is called a problem.

Punctum. The punctum formatum, or generatum in conics, is a point determined by the intersection of a right line drawn through the vertex of the cone, to a point in the plane of the base that constitutes the conic section.

Pyramid, a solid standing on a triangular, square, or polygonal basis, and terminating in a point at the top: or, it is a solid figure, consisting of several triangles, whose bases are all in the same plane and have one common vertex. The superficies of a given pyramid are readily formed by measuring their triangles separately; for their sum added to the area of the base is the surface of the pyramid required. To find the solid content of a pyramid, the area of the base being found, let it be multiplied by the third part of its height, or the third part of the base by the height, and the product will give the solid content. If the solid content of a frustrum of a pyramid is required, let the solid content of the whole pyramid be found; from which subtract the solid content of the part wanting, and the solid content of the frustrum or broken pyramid will remain. Every pyramid is equal to one-third of its circumscribing prism, or that has the same base and height; that is, the solid content of the prism, ABD, No. 2, is equal to one third of the prism, ABFE. Suppose the base A*a*, B*b*, a square, then does the pyramid consist of an infinite number of such squares, whose sides are continually increasing in arithmetical progression, beginning at the vertex or point D; its base, A*a*, B*b*, being the greatest term, and its perpendicular height, CD, the number of all the terms: but the last term multiplied into the number of terms will be triple the sum of all the series, or $\frac{NLL}{3} = S =$ the solid content of the pyramid. All pyramids are in a ratio compounded of their bases and altitudes; so that, if their bases be equal, they are in proportion to their altitudes, &c. Equal pyramids reciprocate their bases and altitudes; that is, the altitude of one is to that of the other as the base of the one is to the base of the other.



Quadrangle, a quadrilateral figure, or one consisting of four sides and four angles.

Quadrant, an arch of a circle containing ninety degrees, or the fourth part of the entire periphery. Sometimes the space, or area, included between this arch and two radii drawn from the centre to each extremity thereof is called a quadrant, or more properly a quadrantal space, being the quarter of an entire circle.

QUADRANTAL TRIANGLE, a spherical triangle, one of whose sides at least is the quadrant of a circle, and one of its angles a right angle.

QUADRATURE denotes the squaring or reducing a figure to a square. The finding of a square, which shall contain just as much surface or area as a circle, ellipsis, or triangle, &c., is the quadrature of a circle, ellipsis, &c.

QUADRILATERAL, a figure whose perimeter consists of four right lines, making four angles; whence it is also called a quadrangular figure. The quadrilateral figures are either a parallelogram, trapezium, rectangle, square, rhombus, or rhomboides.

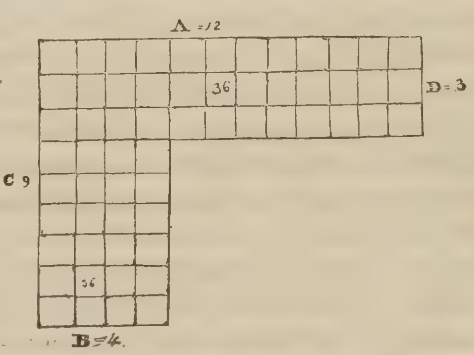
QUINDECAGON, a plane figure with fifteen sides and fifteen angles, which, if the sides be all equal, is termed a regular quindecagon, and an irregular when otherwise. The side of a regular quindecagon inscribed in a circle is equal in power to the half difference between the side of the equilateral triangle and the side of a pentagon inscribed in the same circle; also the difference of the perpendiculars let fall on both sides taken together.

RADIAL CURVES are curves of the spiral kind, whose ordinates (if they may be so called) all terminate in the centre of the including circle, appearing like radii of that circle.

RADIUS, the semidiameter of a circle, or a right line drawn from the centre to the circumference.

RATIO is that relation of homogeneous things which determines the quantity of one from another without the intervention of a third.

RECIPROCAL FIGURES, those which have the antecedents and consequents of the same ratio in both figures. Thus the side A : B :: C : D; or 12 : 4 :: 9 : 3; that is, as much as the side A in the first rectangle is longer than B, so much deeper is the side C in the second rectangle than the side D in the first, consequently the greater length of the one is compensated by the greater breadth or depth of the other; for as the side A is one-fourth longer than C, so B is one-fourth longer than D, and the rectangles of course equal, that is, A×D=B×C, or 12×3=4×9=36. This is the foundation of the theorem, viz., that the rectangle of the extremes is always equal to the means; and, consequently, the reason of the rule of three. Therefore it follows, that if any two triangles, parallelograms, prisms, parallelopipeds, pyramids, cones, or cylinders, have their bases and altitudes reciprocally proportional, those two figures or solids are equal to each other; and, on the other hand, if they are equal, then their bases and altitudes are reciprocally proportional.



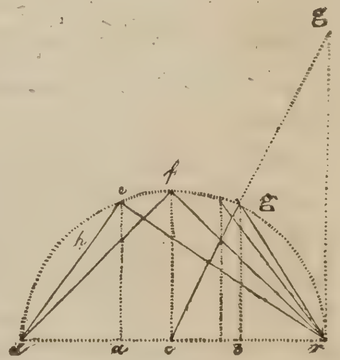
RECTANGLE, the same as a right-angled parallelogram.

RECTANGLED, RECTANGULAR, or RIGHT-ANGLED, appellations given to figures and solids which have one or more right angles: a triangle with one right angle is termed a rectangled triangle; also parallelograms with right angles, squares, cubes, &c., are rectangular. Solids, as cones, cylinders, &c., are also said to be rectangular, with respect to their situation, when their axes are perpendicular to the plane of the horizon.

Rectification is the finding of a right line equal in length to a curve.

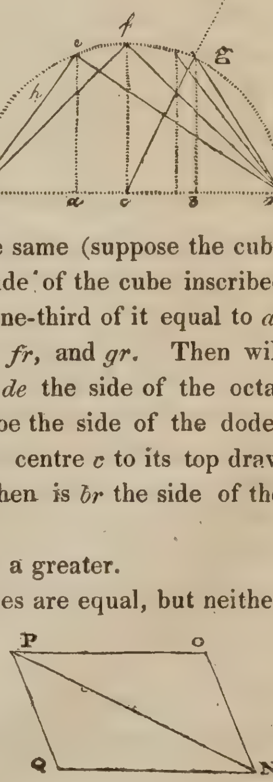
Rectilinear, right-lined, are figures whose perimeter consists of right lines.

Regular denotes any thing agreeable to the rules of art: as a regular figure in geometry, whose sides, and consequently the angles, are equal, and a regular figure with three or four sides is commonly termed a triangle, or square, as all others with more sides are called regular polygons. All regular figures may be inscribed in a circle. A regular solid, called also a platonic body, is that which is terminated on all sides by regular and equal planes, and whose solid angles are all equal. The regular bodies are the five following: first, the tetrahedron, which is a pyramid comprehended under four equal and equilateral triangles: second, the hexahedron, or cube, whose surface is composed of six equal squares: third, the octahedron, which is bounded by eight equal and equilateral triangles: fourth, the dodecahedron, which is contained under twelve equal and equilateral pentagons: fifth, the icosihedron, consisting of twenty equal and equilateral triangles. The proportion of the five regular bodies, the diameter of the sphere being 2. The circumference of the greatest circle is 6.28318. Superficies of the greatest circle 3.14159. Superficies of the sphere 12.56637. Solidity of the sphere 4.18859. Side of the tetrahedron 1.62299. Superficies of a tetrahedron 4.6188. Solidity of a tetrahedron 0.15132. Side of a cube or hexahedron 1.1547. Superficies of the hexahedron 8.000. Solidity of the hexahedron 1.5396. Side of an octahedron 1.41421. Superficies of the octahedron 6.9282. Solidity of the octahedron 1.33333. Side of the dodecahedron 0.71364. Superficies of the dodecahedron 10.51462. Solidity of the dodecahedron 2.78516. Side of the icosihedron 1.05146. Superficies of the icosihedron 9.57454. Solidity of the icosihedron 2.53615. If one of these five regular bodies were required to be cut out of the sphere of any other diameter, it will be as the diameter of the sphere two is to the side of any one solid inscribed in the same (suppose the cube 1.1547), so is the diameter of any sphere (suppose 8) to 9.2376, the side of the cube inscribed in this latter sphere. Let *dr* be the diameter of any sphere, and *da* one-third of it equal to *ab* and *br*. Erect the perpendiculars *ae*, *cf*, and *bg*, and draw *de*, *df*, *er*, *fr*, and *gr*. Then will *re* be as the side of the tetrahedron, *df* the side of the hexahedron, *de* the side of the octahedron. Cut *de* in extreme and mean proposition in *h*, and *dh* will be the side of the dodecahedron. Set the diameter *dr* up perpendicularly at *r*, and from the centre *c* to its top draw the line *cg*, cutting the circle in *g*. Let fall the perpendicular *gb*; then is *br* the side of the icosihedron.

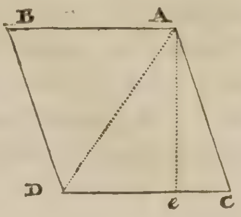


Residual figure, the figure remaining after taking a lesser from a greater.

Rhomboides, a quadrilateral figure whose opposite sides and angles are equal, but neither equilateral nor equiangular; as the figure NOPQ.



Rhombus, an oblique-angled parallelogram, or a quadrilateral figure whose sides are equal and parallel, but the angles unequal, two of the opposite ones being obtuse, and the other two acute, as ABCD. To find the area of a rhombus, upon CD, assumed as a base, let fall the perpendicular A*e*, which is the altitude of the figure; then multiply the base by the altitude, the product will be the area.



Right signifies the same as strait: therefore a strait line is a right line.

Rotation, a term applied to the circumrotation of any surface round a fixed and immovable line, which is called the axis of its rotation; and by such rotations it is that solids are conceived to be generated.

Scalence, or **scalenous triangle**, a triangle whose sides and angles are unequal.

Scholium, a note, annotation, or remark, occasionally made upon some passage, proposition, &c. This term is used in geometry and the mathematics, where, after demonstrating a proposition, it is customary to point out how it might be done some other way; or give some advice or precaution, in order to prevent mistakes, or add some particular use upon application thereof.

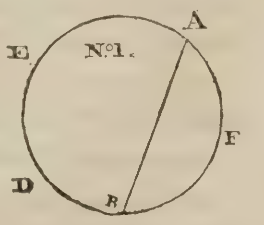
Screw, a screw is a cylinder cut into several concave surfaces, or rather a channel or groove made in a cylinder, by carrying on two spiral planes the whole length of the screw, that they may be always inclined equally to the axis of the cylinder in their whole progress, and also always inclined to the base of it in the same angle. The screw may also be considered as a wedge carried round a cylinder, which is called the arbor of the screw; the wedge so carried onwards making the thread of the screw.

Secant is a line that cuts another, or divides it into two parts.

Section denotes a side or surface appearing of a body or figure cut by another; or the parts where lines, planes, &c., cut each other. The common section of two planes is always a right line, being the line supposed to be drawn on one plane by the section of another, or by its entrance into it.

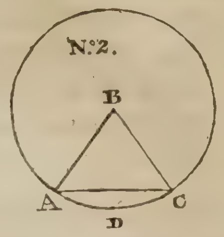
Sector is part of a circle, comprehended between two radii and the arch; or it is a mixed triangle formed by two radii and the arch of a circle.

Segment of a circle, that part of the circle contained between a chord and an arch of the same circle. The portion AFB, and the chord AB, is a segment of the circle ABFD. It is evident every segment of a circle must either be greater or less than a semicircle, the greater part of the circle cut off by a chord; that is, the part greater than a semicircle is called the greater segment, as ABED; and the lesser part, or part less than a semicircle, the lesser segment, as AFB. It appears that the area of the sector ABCD, No. 2, is produced by multiplying half of the arch into the



radius; and likewise the area of the segment ABC is found by subtracting from the area of the sector the area of the triangle ABC.

Segment of a sphere is a part of a sphere terminated by a portion of its surface, and a plane which cuts it off, passing somewhere out of the centre, being more properly called the section of a sphere: the base of such segment, it is evident, must always be a circle.



Semi-circle, half a circle, or that figure comprehended between the diameter of a circle and half the circumference.

Semi-diameter, half the diameter, or a right line drawn from the centre of a circle, or sphere, to its circumference, being the same with what is otherwise called the radius.

Semi-ordinate, the half of an ordinate.

Sexangle, a figure having six sides, consequently six angles.

Similar figures, such as have their angles respectively equal, and the sides about the equal angles proportional.

Solid. A solid is defined to be the third species of magnitude, or that which has three dimensions, viz., length, breadth, depth or thickness. A solid may be conceived to be formed by the revolution or direct motion of a superficies of any figure whatever, and is always terminated or contained under one or more planes or surfaces, as a surface is under one or more lines. Solids are divided into regular and irregular. The regular solids are those terminated by regular and equal planes, and are only five in number, viz., the tetrahedron, which consists of four equal triangles; the cube, or hexahedron, of six equal squares; the octahedron, of eight equal triangles; the dodecahedron, of twelve; and the icosihedron, of twenty equal triangles.

The irregular solids are almost infinite, comprehending all such as do not come under the definition of regular solids; as the sphere, cylinder, cone, parallelogram, prism, parallelopiped, &c.

Solid angle is that formed by three or more planes meeting in a point.

Solid problem is one which cannot be geometrically solved unless by the intersection of a circle and a conic section: or, by the intersection of two other conic sections besides the circle: as to describe an isosceles triangle on a given right line, whose angle at the base shall be triple to that at the vertex. This will help to inscribe a regular heptagon in a given circle, and may be resolved by the intersection of a parabola and a circle. This problem also helps to inscribe a nonagon in a circle, and may be solved by the intersection of a parabola and an hyperbola between its assymptotes, viz., To describe an isosceles triangle, whose angle at the base shall be quadruple of that at the vertex. And such a problem as this has four solutions, because two conic sections can cut one another but in four points.

Solidity. The solidity of a body denotes the quantity or space contained in it, and is called also its solid content.

Solution. The answering a question, or resolving any problem proposed.

Space denotes the area of any figure, or that which fills the interval or distance between the lines that terminate it.

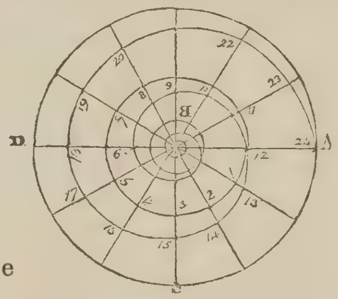
Specific gravity is that by which one body is heavier than another of the same dimension, and is always as the quantity of matter under that dimension.

Sphere is a solid contained under one uniform round surface, such as would be formed by the revolution of a circle about a diameter thereof, as an axis. The circle revolving about the diameter will generate a sphere, whose surface will be formed by the circumference of the circle. The centre and axis of a sphere are the same as the centre and diameter of the generating circle: and as a circle has an indefinite number of diameters, so a sphere may be considered as having also an indefinite number of diameters, round any one of which the sphere may be conceived to be generated. All spheres are to one another as the cubes of their diameters. The surface of the sphere is equal to four times the area of one of its great circles. To find the superficies of any sphere, let the area of a great circle be multiplied by four, and the product will be the superficies. The solidity of a sphere is equal to the surface multiplied into one-third of the radius: or, a sphere is equal to two-thirds of its circumscribing cylinder, having its base equal to a great circle of the sphere. A sphere is equal to a cone whose height is equal to the semi-diameter of the sphere, and its base equal to the superficies of the sphere, or to the area of four great circles of the sphere, or to a circle whose radius is equal to the diameter of the sphere.

Spherics is that part which treats of the position and mensuration of arches of circles described on the surface of the sphere.

Spheroid, a solid approaching to the figure of a sphere. The spheroid is generated by the entire revolution of a semi-ellipsis about its axis.

Spiral, a curved line of the circular kind, which, in its progress, recedes from its centre. A spiral is generated by having one end fixed in B upon the line ABD, and equally moved round, so as with the other end A to describe the periphery of a circle; and, at the same time, a point be conceived to move forward equally from B towards A in the right line BA, so as that the point describes such line, while the line generates the circle: then will the point, with its two motions, describe the curved line B 1, 2, 3, 4, 5, &c., which is called the helix or spiral line; and the plane space, contained between the spiral line and the right line BA, is called the spiral space. If the point B was to be conceived to move twice as slow as the line AB, so that it shall get but half-way along the line BA, when that line shall have formed the circle; and if then you imagine a new revolution to be made of the line carrying the point, so that they shall end their motion at last together, there will be formed a double spiral line, and the two spiral spaces, as you see in the figure. From the principle of this curve, the following corollaries may be drawn. First, the lines B 12, B 11, B 10, &c., making equal angles with the first and second spiral (as B 12, B 10, B 8, &c.), are in arithmetical proportion. Second, the lines B 7, B 10, &c., drawn any how to the first spiral, are to one another as the arches of the circle intercepted between BA and those lines. Third, any lines drawn from B to the second spiral, as B 18, B 22, &c., are to each other as the aforesaid arches, together with the whole periphery added on both sides. Fourth, the first spiral space is to the first circle as one to three. And fifth, the first spiral line is equal to half the periphery of the first circle; for the radii of the sectors, and consequently the arches, are in a simple arithmetic progression, whilst the periphery of the



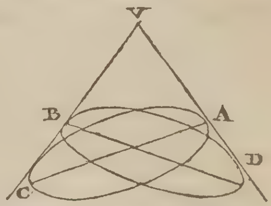
circle contains as many arches equal to the greatest; therefore, the periphery to all those arches is to the spiral lines as two to one.

SQUARE, a quadrilateral figure, both equilateral and equiangular. To find the area of a square, obtain the length of one side; multiply this by itself, and the product is the area of the square.

STEREOMETRY, that part of geometry which teaches how to measure solid bodies, that is, to find the solidity of solid contents of bodies, as globes, cylinders, cubes, &c.

STEREOTOMY, the art of cutting solids, or making sections thereof.

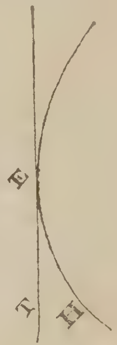
SUBCONTRARY POSITION is when two similar triangles are so placed as to have one common angle V at the vertex, and yet their bases not parallel. If the scalenous cone, BVD, be so cut by the plane CA, as that the angle at C = the angle at D, the cone is then said to be cut contrarily to its base BD.



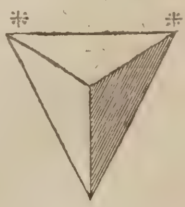
SUBTEUSE, the same with the chord of an arch. The subteuse of an angle is a right line, supposed to be drawn between the two extremities of the arch that measures the angle.

SUPERFICIES, or SURFACE, a magnitude considered as having two dimensions, or extended in length and breadth, but without thickness or depth. In bodies, the superficies is all that presents itself to the eye. A superficies is chiefly considered as the external part of a solid. When we speak of a surface simply, and without any regard to body, we usually call it figure. The several kinds of superficies are as follow. Rectilinear superficies that are comprehended between curved lines; plane superficies are those which have no inequality, but lie evenly between their boundary lines; convex superficies is the exterior part of a spherical or spheroidal body. The measure or quantity of a superficies, or surface, is called the area thereof.

TANGENT is defined in general to be a right line, ET, which touches any arch of a curve, HE in E, in such a manner that no right line can be drawn through E between the right line ET and the arch EH, or within the angle HET that is formed by them. The tangent of an arch is a right line drawn perpendicularly from the end of a diameter, passing to one extremity of the arch, and terminated by a right line drawn from the centre through the other end of the arch, and called the secant. The cotangent of an arch is a tangent of the complement of the arch. The tangent of a curve is a right line which only touches the curve in one point, but does not cut it.



TETRAHEDRON, one of the five regular or Platonic bodies or solids, comprehended under four equilateral and equal triangles.



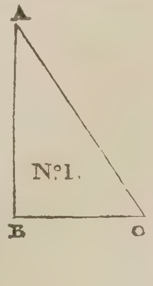
TETRAGON, a general name for any four-sided figure, as a square, a parallelogram, rhombus, or trapezium.

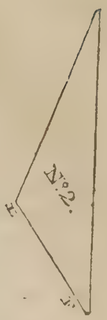
THEOREM, a speculative proposition, demonstrating the properties of any subject. Theorems are either universal, which extends to any quantity without restriction, universally; as this, that the rectangle of the sum and difference of any two quantities, is equal to the difference of their squares: or particularly, which extends only to a given quantity, as in an equilateral right-lined triangle, each of the angles is sixty degrees. A negative theorem is that which expresses the impossibility of any assertion; as, the sum of two biquadrate numbers cannot make a square number. A local theorem is that which relates to a surface; as, the triangles of the same base and altitude are equal. A plane theorem is that which either relates to a rectilinear surface, or to one terminated by the circumference of a circle; as, that all angles in the same segment of a circle are equal. And a solid theorem is that which considers a space terminated by a solid line; that is, by any of the three conic sections.

THEOREM RECIPROCAL is one whose converse is true, viz., if a triangle have two equal sides, it must have two equal angles; the converse of which is likewise true; if it has two equal angles, it must have two equal sides.

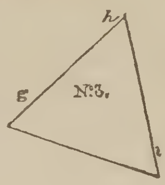
TRAPEZIUM, a plane figure, contained under four unequal right lines. First, Any three sides of a trapezium taken together are greater than the third. Second, The two diagonals of a trapezium divide it into four proportional triangles. Third, If two sides of a trapezium be parallel, the rectangle under the aggregate of the parallel sides and one half their distance is equal to it. Fourth, If a parallelogram circumscribes a trapezium, so that one of the sides of the parallelogram be parallel to a diagonal of the trapezium, that parallelogram will be double the trapezium. Fifth, If any trapezium has two of its opposite angles, each a right angle, and a diagonal be drawn joining these angles; and if from the other two angles be drawn two perpendiculars to that diagonal, the distances from the feet of these perpendiculars to those right angles, respectively taken, will be equal. Sixth, If the sides of a trapezium be each bisected, and the points of bisection be joined by four right lines, these lines will form a parallelogram, which will be half of the trapezium. Seventh, If the diagonals of a trapezium be bisected, and a right line join the points, the aggregate of the squares of the sides are equal to the aggregate of the squares of the diagonals, together with four times the square of the right line joining the point of bisection. Eighth, In any trapezium, the aggregate of the diagonals is less than the aggregate of the four right lines drawn from any point (except the intersection of the diagonals) within the figure.

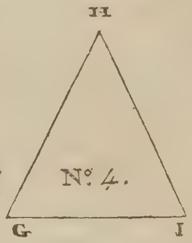
TRIANGLE, a figure of three sides and three angles. Triangles are either plane or spherical. A plane triangle is contained under three right lines; and a spherical one is a triangle contained under three arches of great circles of the sphere. Triangles are denominated from their angles, right, obtuse, and acute. A right-angled triangle is that which has one right angle, as ABC, No. 1. An obtuse-angled triangle is such as has one obtuse angle, as DEF, No. 2. An acute-angled triangle is that which has all its angles acute, as *ghi*, GHI, No. 3 and 4. Any triangle that is not right-angled, is called oblique-angled or amblygonial.





An equilateral triangle is that which has all its sides equal; as *ghi*, No. 3. An isosceles triangle is one that has only two sides equal; as GHI, No. 4. And a scalenous triangle has no two sides equal; as DEF, No. 2. Triangles in the same base, and having the same height or place between the same parallels, are equal: also triangles on equal bases and between the same parallels are equal. If a perpendicular be let fall upon the base of an oblique-angled triangle, the difference of the squares of the sides is equal to the double rectangle under the base, and the distance of the perpendicular from the middle of the base. The side of an equilateral triangle, described in a circle, is in power triple the radius. The sides of a triangle are cut proportionably by a line drawn parallel to its base. A whole triangle is to a triangle cut off by a right line drawn parallel to its base, as the rectangle under the cut side is to the rectangle of the two other sides. In a right-angled triangle, a line drawn from the right angle at the top, perpendicular to the hypotheneuse, divides the triangle into two other right-angled triangles, which are similar to the first triangle, and to one another. In every right-angled triangle, the square of the hypotheneuse is equal to the sum of the squares of the other two sides. If any angle of a triangle be bisected, the bisecting line will divide the opposite side in the same proportion as the legs of the angle are to one another. Every triangle is half of a parallelogram of the same base and height.

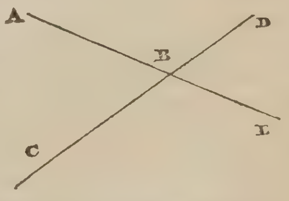




TRISECTION, the division of a figure into three, or the division of an angle into three equal parts. The trisection of an angle geometrically is one of those great problems whose solution has been so much sought by mathematicians for these two thousand years, being in this respect on a footing with the quadrature of the circle, and the duplicature of the cube.

TRUNCATED is an appellation given to such figures as have, or seem to have, the points cut off, as a truncated cone, pyramid, &c.

VERTEX, the upper part or top, as the vertex of a cone or pyramid, &c., being the top of those figures. The vertex of an angle is the angular point, and the angles which, being opposite and touch only in the angular point, are called vertical angles: such are the angles ABC, and DBE, wherein the sides AB and CB are only continuations of the legs of the other, BE and BD; such angles are demonstrated to be equal. The vertex of any plane figure is the angle opposite to the base; and the vertex of a curve is the point from which the diameter is drawn, or the intersection of the diameter and curve.



VERTICAL PLANE is a plane passing through the vertex of the cone, and parallel to any conic section. And a vertical line is a right line drawn on the vertical plane, and passing through the vertex of the cone.

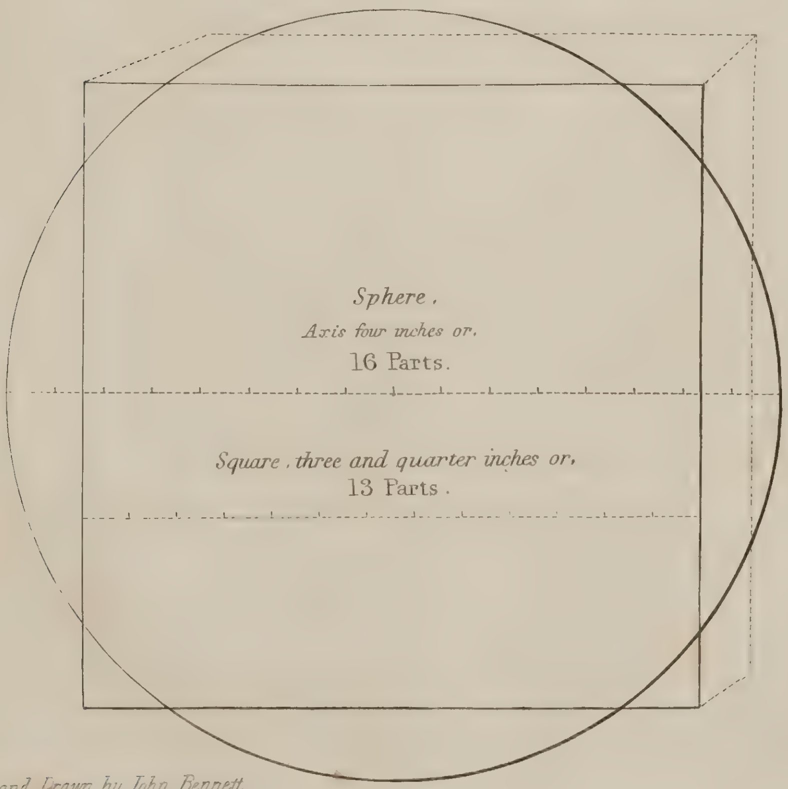
VOLUTE, the spiral.—See SPIRAL.

WEDGE, one of the mechanical powers. The wedge is a triangular prism, whose bases are equilateral acute-angled triangles.

THE
SOLIDITIES OF THE
SPHERE, AND CUBE,
IDENTIFIED.

FIG. 2.

The axis of the Sphere, being divided into sixteen equal parts; thirteen of those parts form one side of the Cube, containing the same solidity.



Invented and Drawn by John Bennett.

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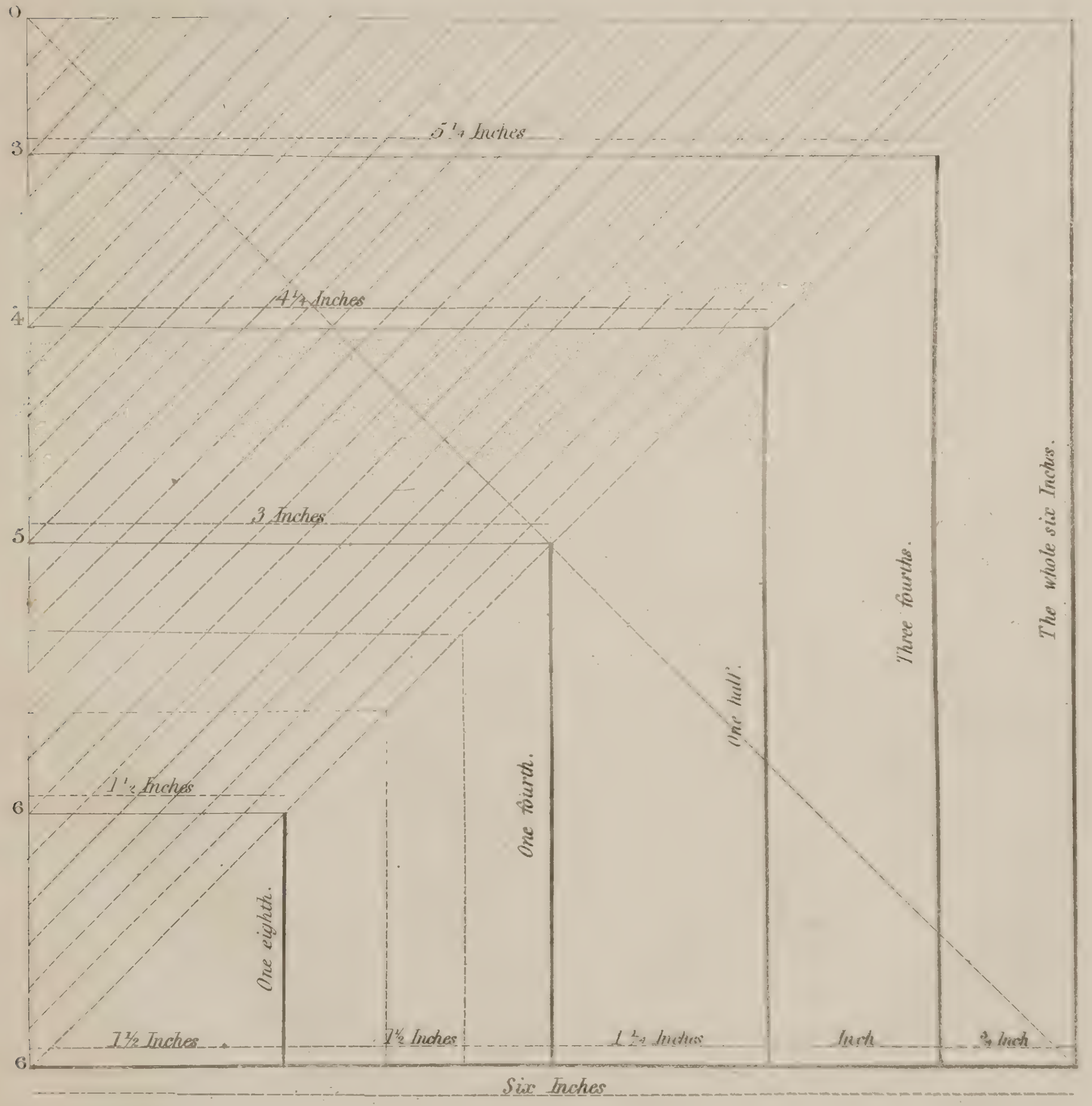
THE SQUARE, Nº 1.

Divided into equal parts, viz. one eighth, one fourth, one half, and three fourths, by the Diagram six inches square.

FIG. 3.



Drawn by John Bennett

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1836.

DESCRIPTION OF THE PLATES.

PLATE I.

The Circle, Square, and Triangle, are the primitive or leading figures in the science of geometry, unto which all others are connected or related throughout the whole of their various transmutations. They are the only emblems that may be termed regular; for, from their nature, they can imply no other sensible representation. In them also are generated and established the extensive and comprehensive system from which arises the production of every other geometrical appearance, whatever be the qualified external formation.

PLATE II.

The Sphere and Cube. The axis of the sphere being equally divided into sixteen parts, thirteen of those parts compose the side of the cube whose solidity or admeasurement will be the same, and consequently of equal gravitation, provided the two bodies be fabricated from the same material.

PLATE III.

The square is here represented divided into aliquot parts, viz. the fourth, half, and three-fourths, by equal diagonal lines. No figure, or representation, has ever yet been discovered to perform this operation exactly: as this method demonstrates. The dimension of the square in the division is not positively requisite, and may be or not dispensed with, as the case may happen; for, in the event of fractional parts arising from the divisions by common arithmetic, many figures are used, and the solution less likely from that circumstance alone to be correct. Allowing, therefore, a square to be divided whose given dimension was five inches and three-quarters, or five feet nine inches and a quarter, it matters not; the method shown here is simply to lay the squared own to the scale of the common two-foot rule, and, as soon as the divisional parts are produced, their quantity or dimension will be ascertained.

PLATE IV.

This square discloses the method of taking half of another square, and the same to be precisely square. Halving the square with parallel lines will not effect squares, but rectangles. Upon this figure, A, the triangle, which is one-fourth of the square, added to B below, makes half. A line drawn parallel to B through the centre would form the square into two parallelograms of similar areas.

PLATE V.

Corresponding areas of the square and circle. The divisions shown upon this diagram would be adapted for any other, and the dimension of the square or circle need not be known: it will always be found sufficient to place the figure at such convenient scale as to obtain a correct solution; and the larger the better, for the truth will be manifest upon the face of it.

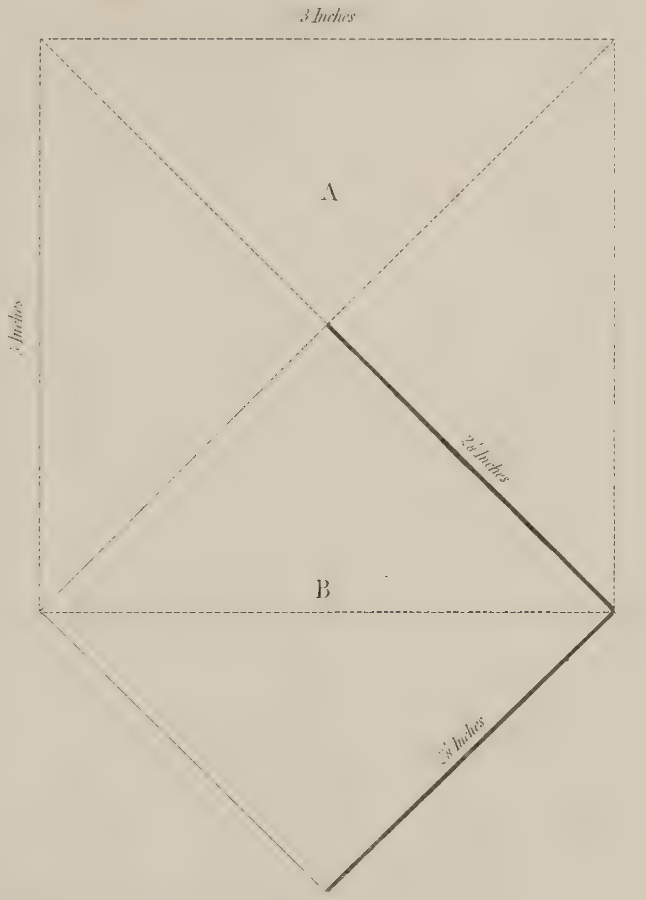
SQUARE.

No. 2.

A. *The square being given whose side is 3 inches, required the half of the same, and that to be square also.*——

B. *The half whose side is $2\frac{1}{8}$ inches.*——

FIG. 4.



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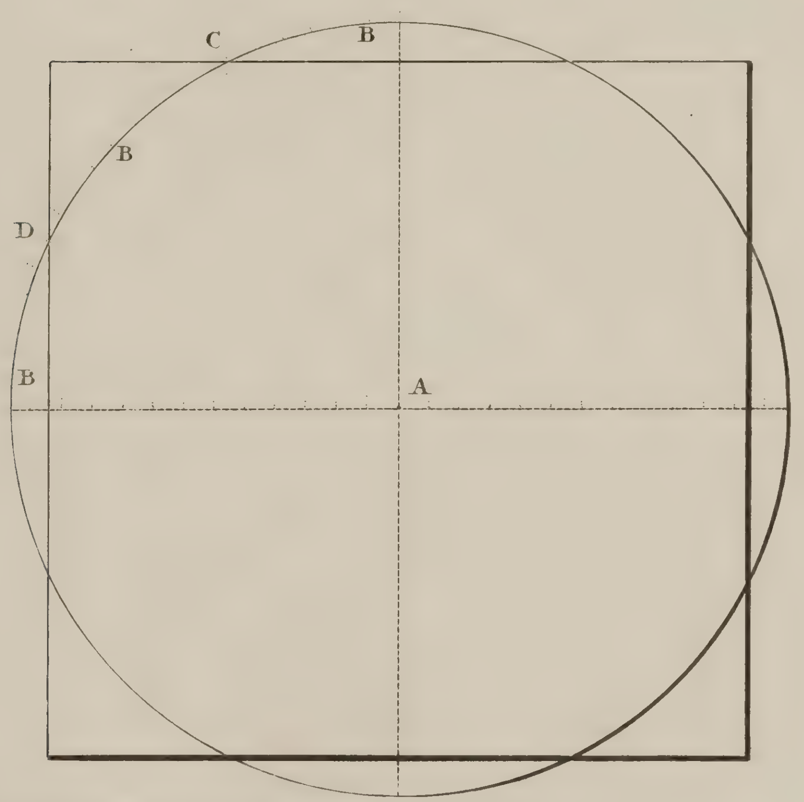
CORRESPONDING AREAS
OF THE
SQUARE AND CIRCLE.

The transverse A of the Circle, being divided into twenty six equal parts; twenty one of those parts are found to occupy one fourth of the circle, as shown at B; and from the divisions C, and D, the sides of the square are found, and drawn, containing the Area of the circle.

FIG. 5.



Invented and drawn by John Bennett, 3rd Feby 1836.

This problem has remained altogether in obscurity; although, rewards were offered by Charles 5th of one hundred thousand crowns: and the States of Holland a similar sum, to any person effecting it: but, it does not appear ever to have been performed; being never published.

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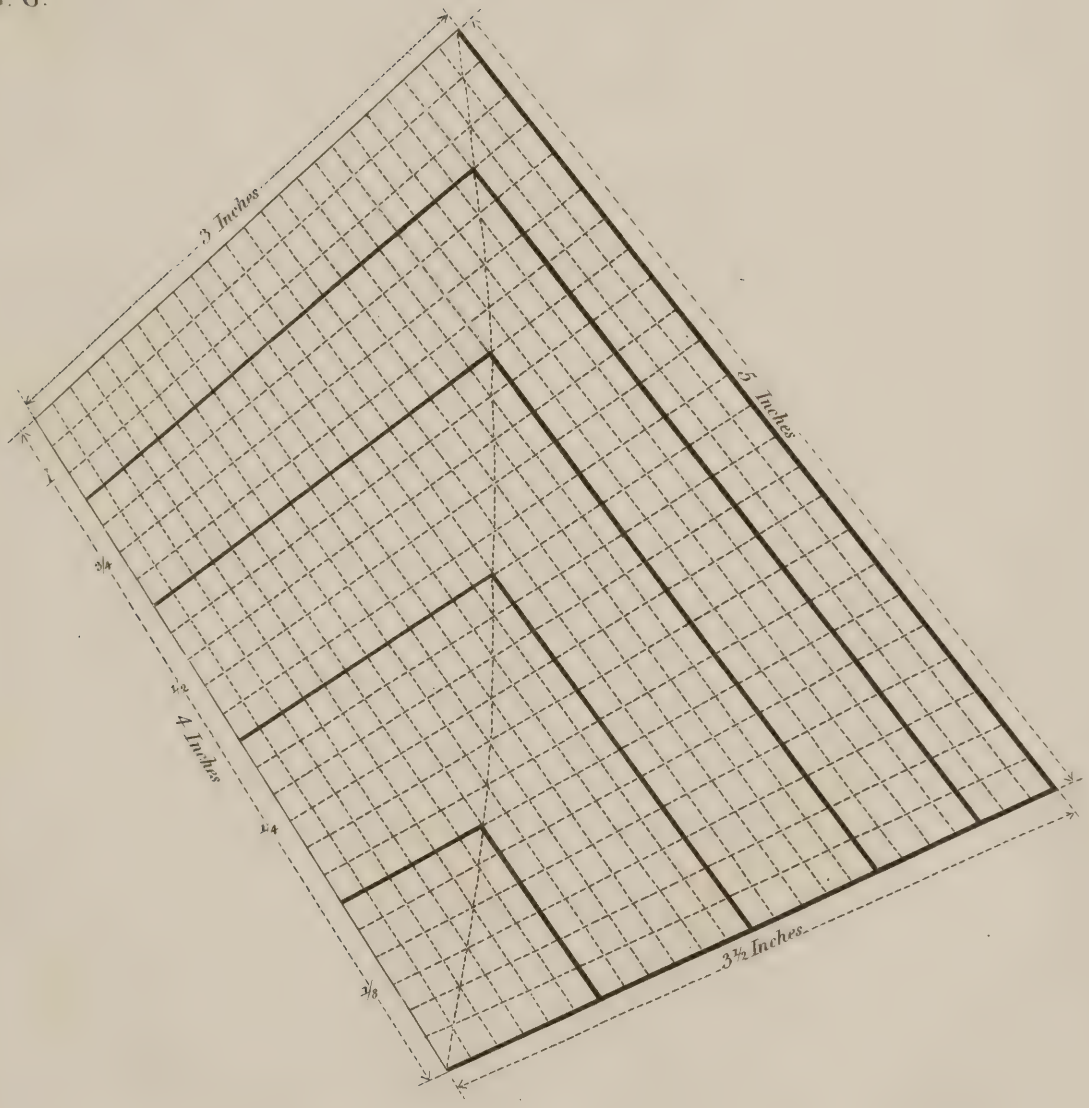
1836

TRAPEZIUM.

FIG. 6.



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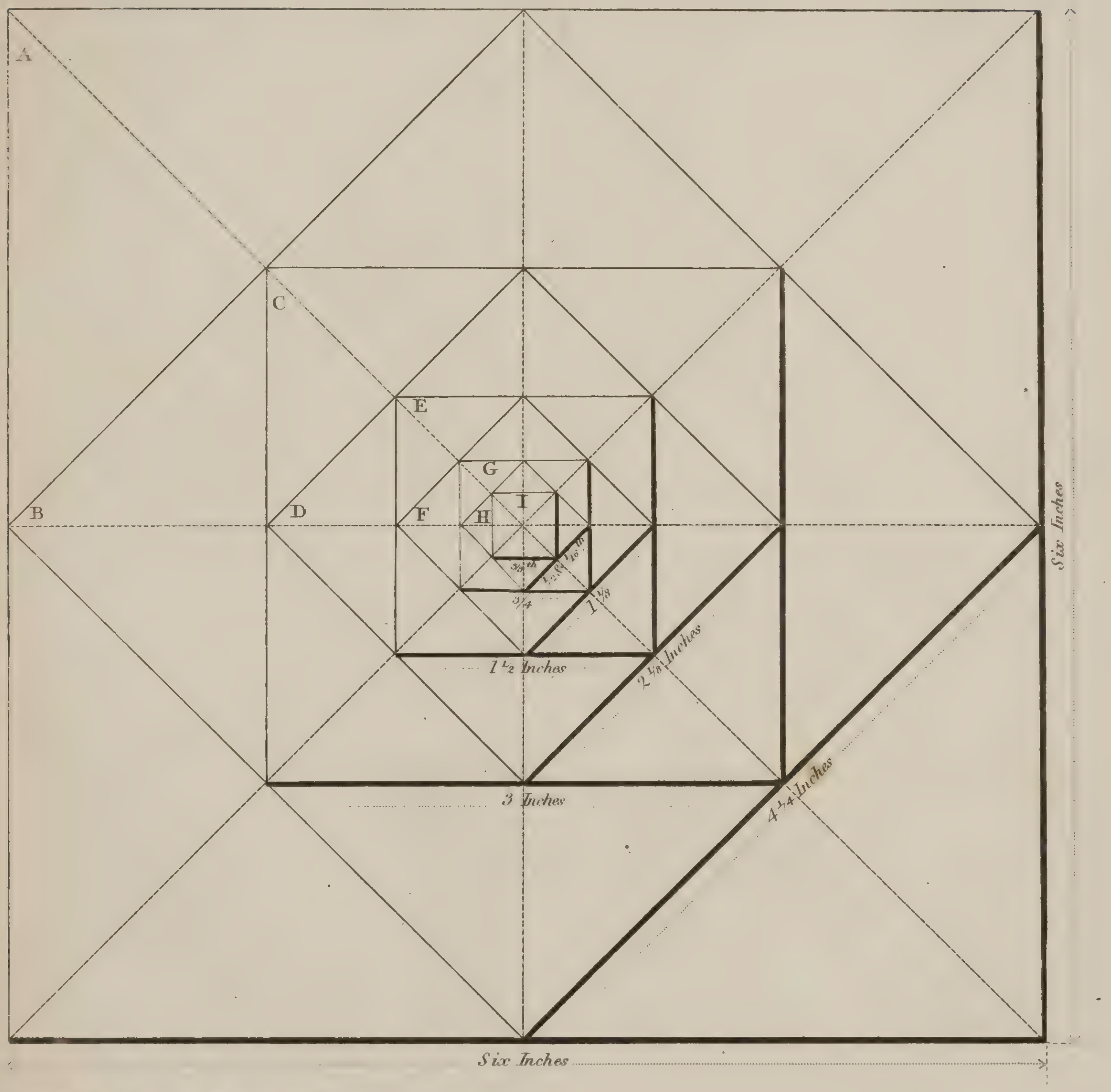
1836

THE SQUARE.

B *the half of* A; C *the half of* B, D *the half of* C, &c.

Again, Triangle B, *the half of* A *and so on for the other.*

FIG. 7.



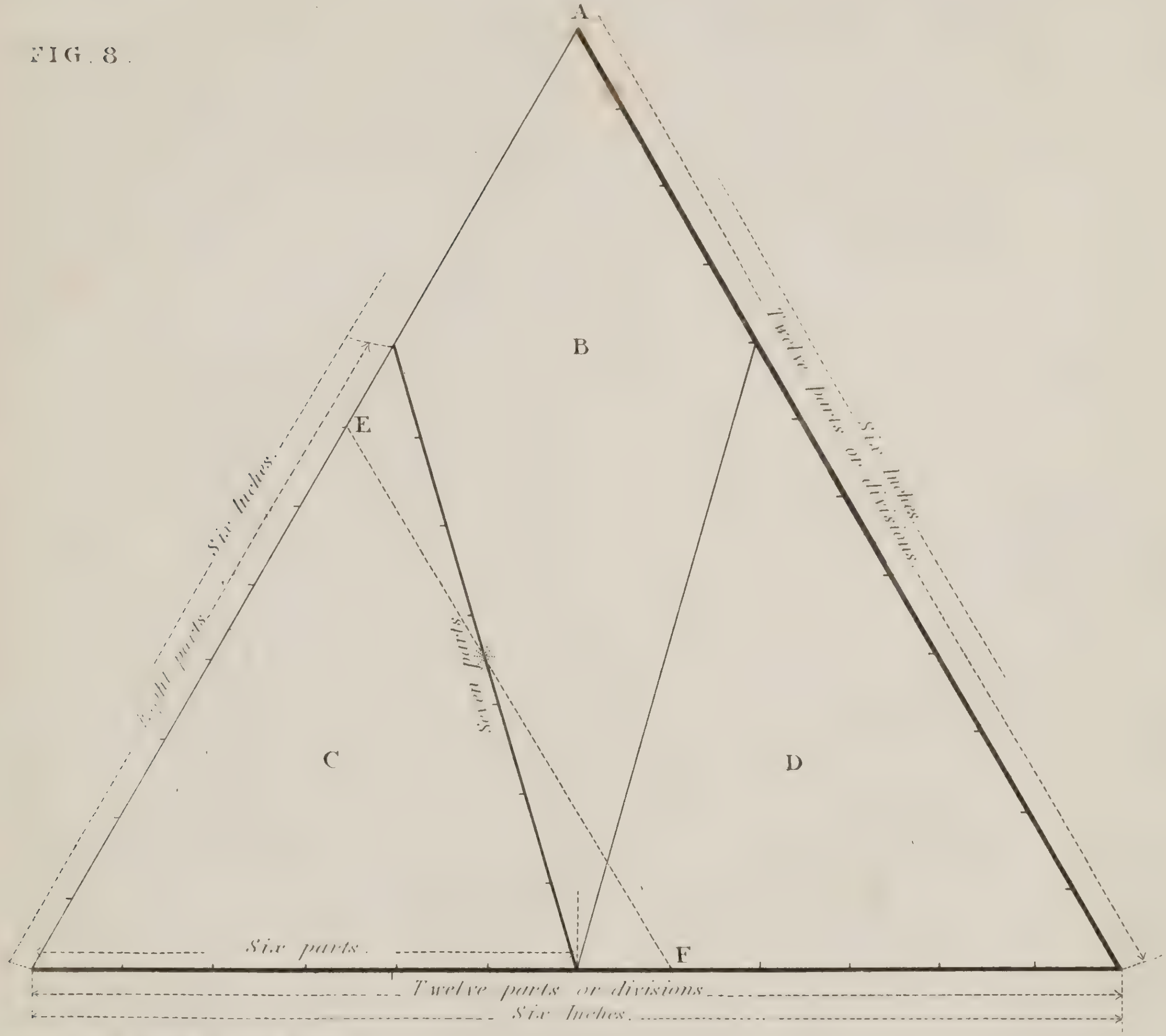
Designed and Drawn by John Bennett.

London. John Bennett 4 Three Tun Passage Ivy Lane Paternoster Row.
[illegible]

TRIANGLE.

A. *The Triangle, divided into three parts* **B. C. D.** *of equal Areas,*
and the dotted line **E F.** *equalises.*

FIG. 8.



The trisection, or division of the triangular figure, into three equal parts.
"The trisection of an angle Geometrically, is one of those great problems whose solution has been so much sought by mathematicians for these two thousand years; being in this respect, on a footing with the quadrature of the Circle; and the duplicature of the cube". New Dictionary Arts and Sciences, Published by a society of Gentlemen, 1754.

PLATE VI.

The trapezium, being an unequal-sided figure, is shown divided in the same manner as the square, figure 3. Although this figure is untrue, this can make no difference in the correct quantities, notwithstanding the dissimilarity in the shape of the smaller divisions.

PLATE VII.

The square diagonally divided from centre points. B is precisely half the area of A, and C half that of B, &c. This figure shows also the complement of the triangle with that of the square.

PLATE VIII.

The triangle divided into three parts of equal quantities this figure clearly demonstrates. It is evident, from the immense sums of money that were formerly offered by kingdoms for the performance of this very difficult problem, that its solution was held by the ancients of infinite importance. It must be particularly observed that there can be no diagram throughout the whole science which will serve to exhibit the value of this discovery so much as the triangle, in the performance of abstruse questions. The trapezium B, and the unequal triangles C and D, have methodically the same areas ; the dotted line EF equalises the triangle C, and the same with D. B, having a different figure, contains the same area as the true figure C, notwithstanding.

PLATE IX.

The rhombus divided into parts, &c. This figure is similar to the square, but differently posited, for the greater facility in comprehending its nature and effects.

PLATE X.

This blended figure, which is composed of circular and angular lines, being purely speculative, has no particular qualities to enumerate ; it may be considered merely as a reference for the formation of others from its diversified positions. In many instances its uses may be imagined from events which frequently occur in the absence of matter to fill up ornamental spaces, and in the arrangement of component parts, in order that the whole should harmonise together. It may likewise be found a desirable reference in panelling out ceilings, laying down embellished floorings, devising pleasure-grounds to noblemen's mansions, and inlaying cabinet work.

PLATE XI.

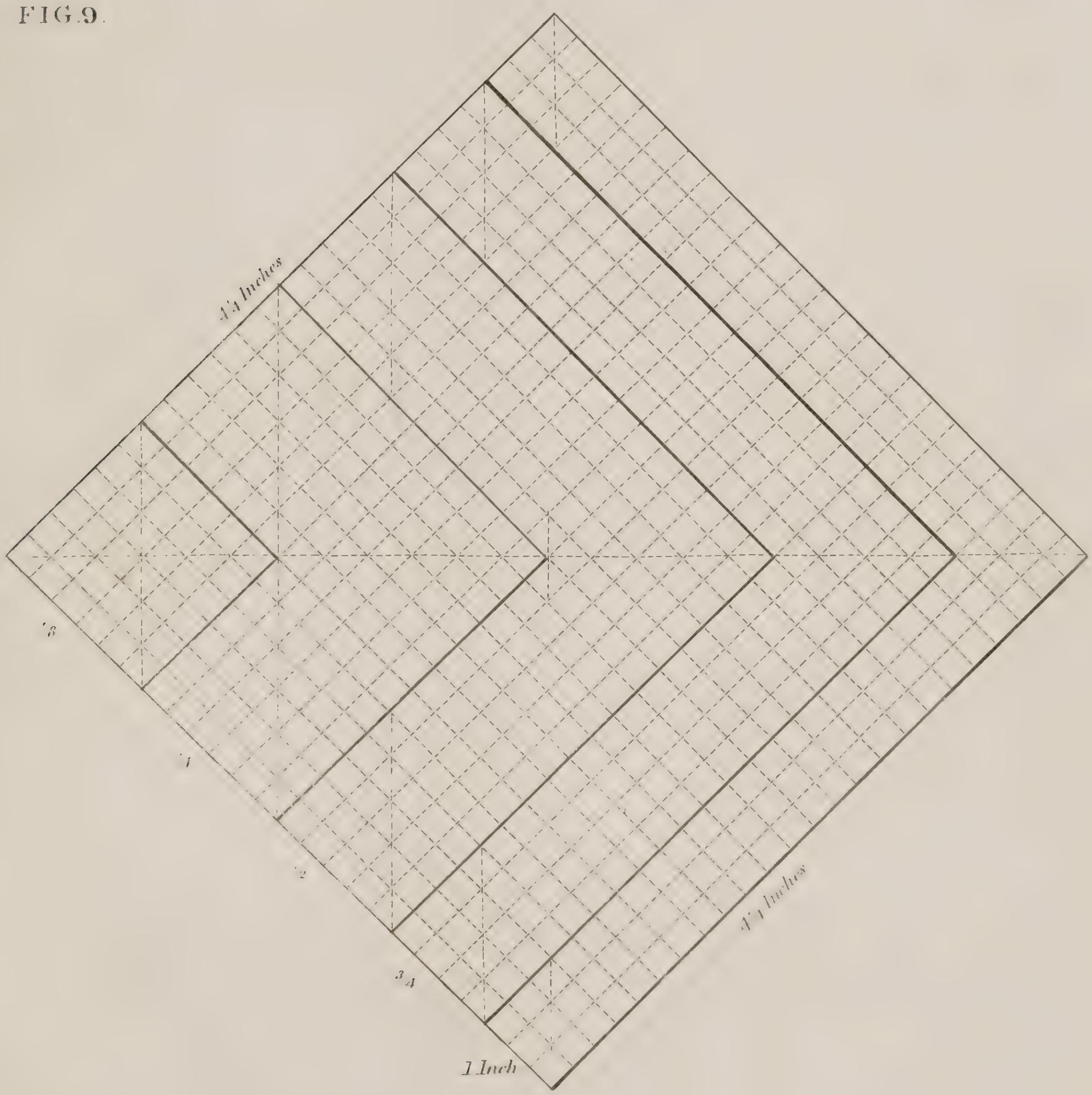
This figure shows that the invention of dividing the square, as shown by the plate 3, is not confined to regular, perfect, or true figures, but can be extended to any other, however disproportionate they may happen to be ; for herein it is made very evident that neither disposition nor quality can alter the verity of this system, for, as they are divided, the result will be certain to follow ; and, although the figure itself is altogether untrue, the divisions will be found mathematically exact.

RHOMBUS N°. 1.

THE QUARRY OF DIAMOND.

Divided into equal parts, viz. 1/8, 1/4, 1/2, 3/4, & 1 Inch.

FIG. 9.



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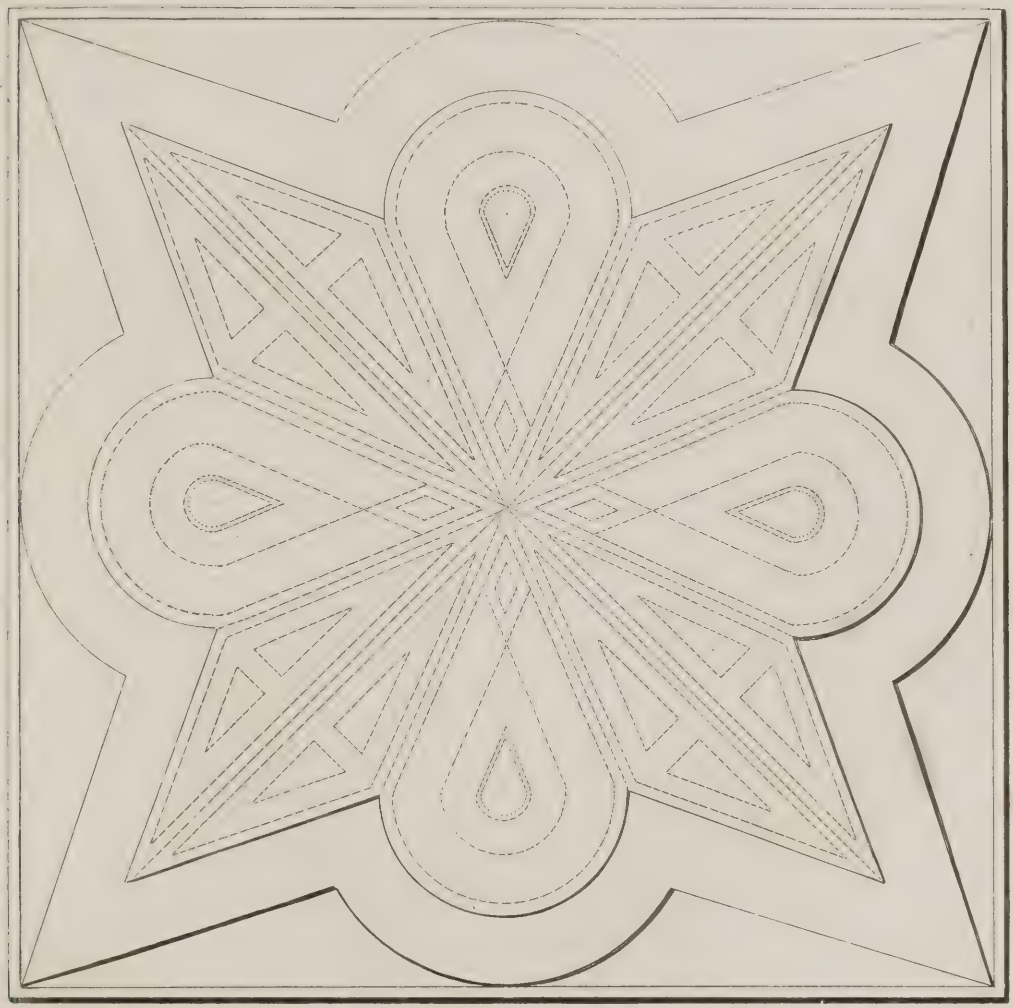
London; John Bennett, Paternoster Row. 18. 6

COMPOUND OF
CIRCULAR AND ANGULAR.

FIG.10.



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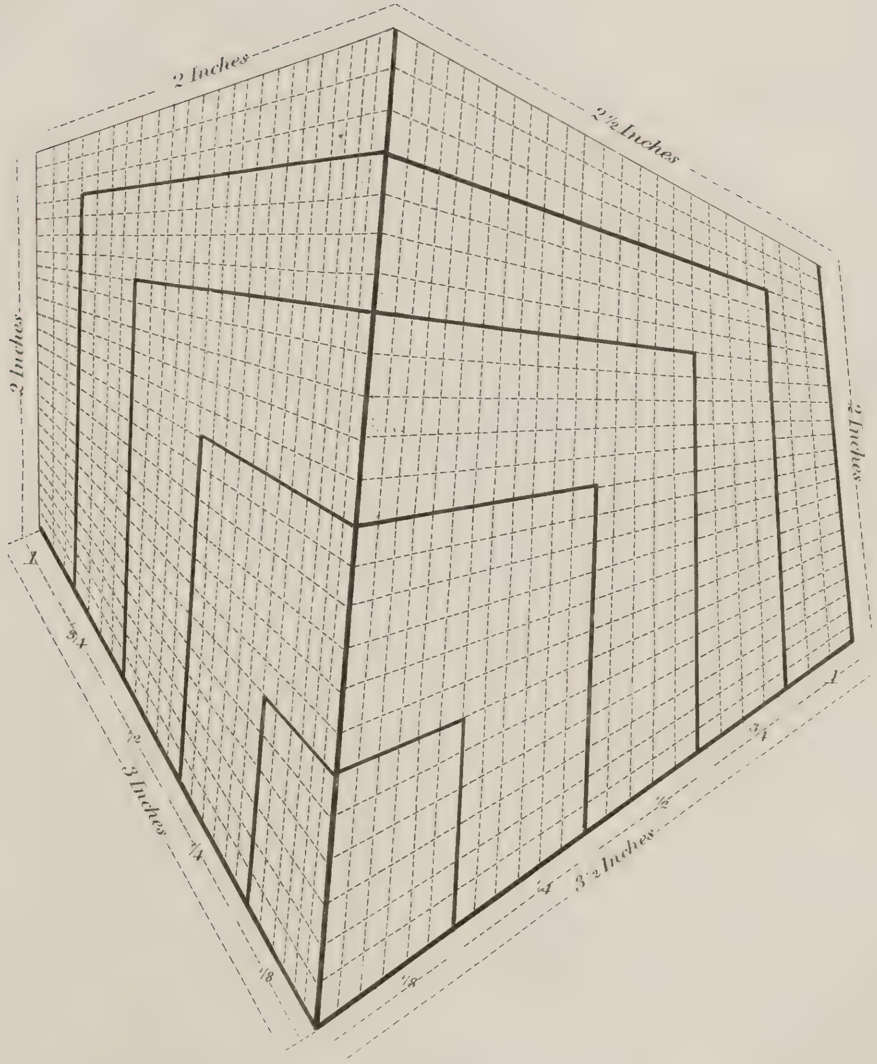
London. John Bennett 4.Three

COMPOUND

FIG. 11.

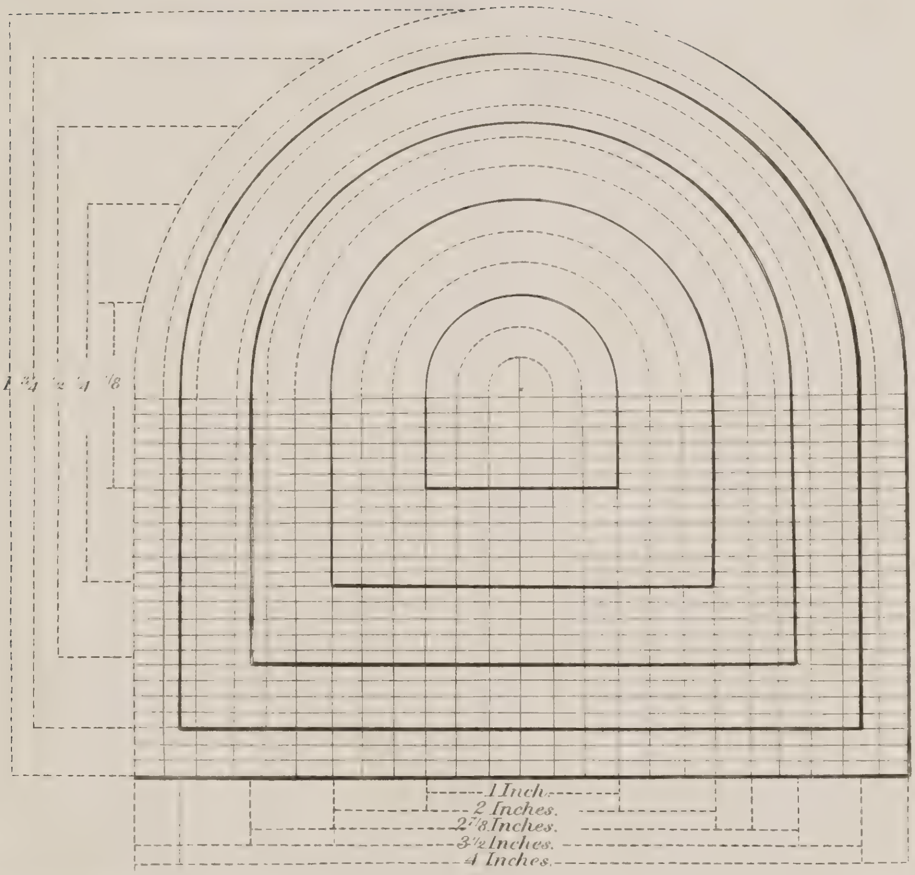


[illegible] [illegible] drawn by John [illegible]

[illegible]

HYPERBOLA

FIG.12.

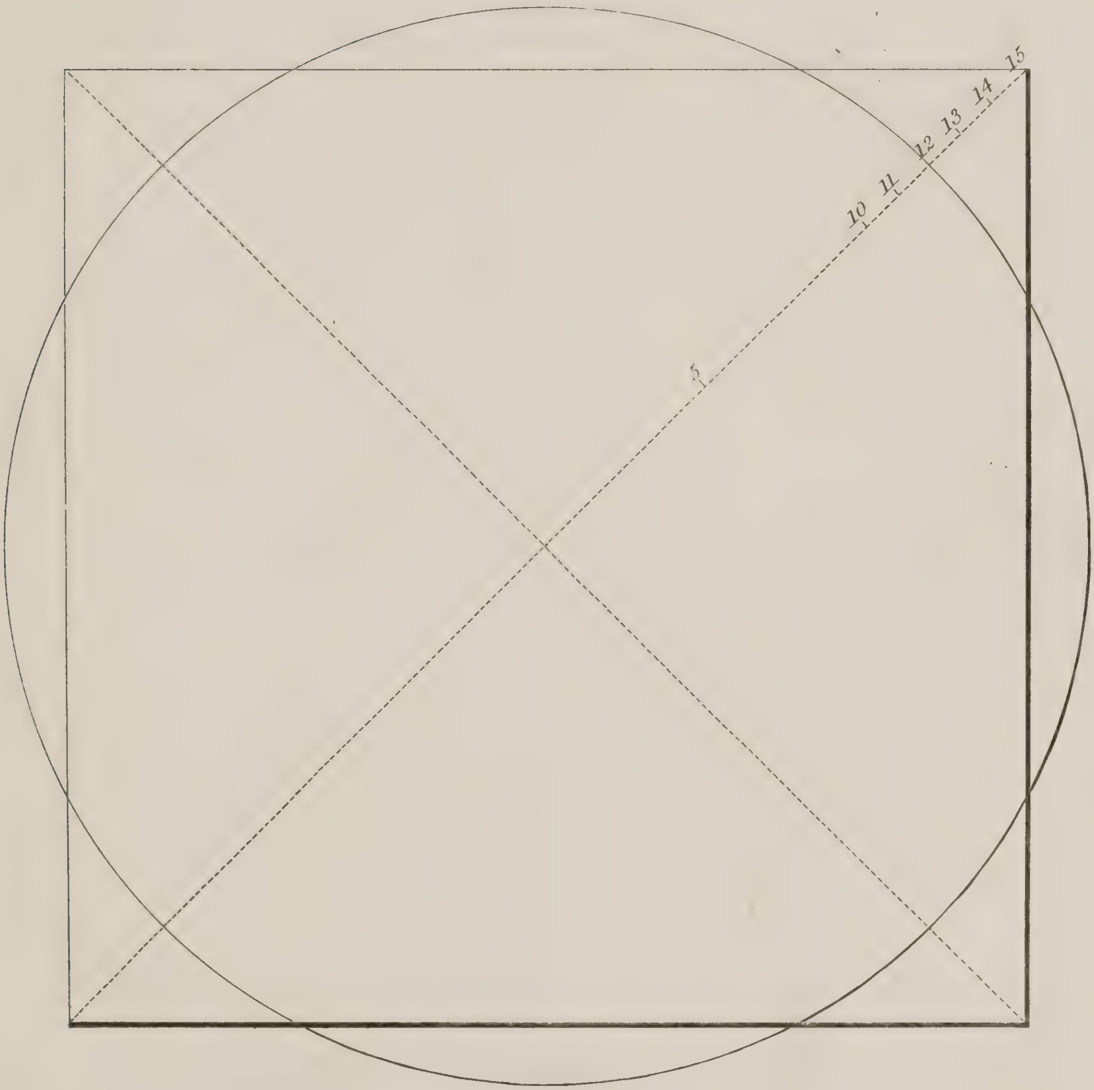


[illegible]

SQUARE & CIRCLE

From a given square, inscribe a circle, containing the same area.

FIG. 13.



ed and Drawn by John Bennett

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SQUARE

Q. *A diagonal line being given as* A,B, *required the side of a square to correspond.*
A. *Divide the diagonal into 17 equal parts, as shown 12 of which form the square.*

FIG. 14.

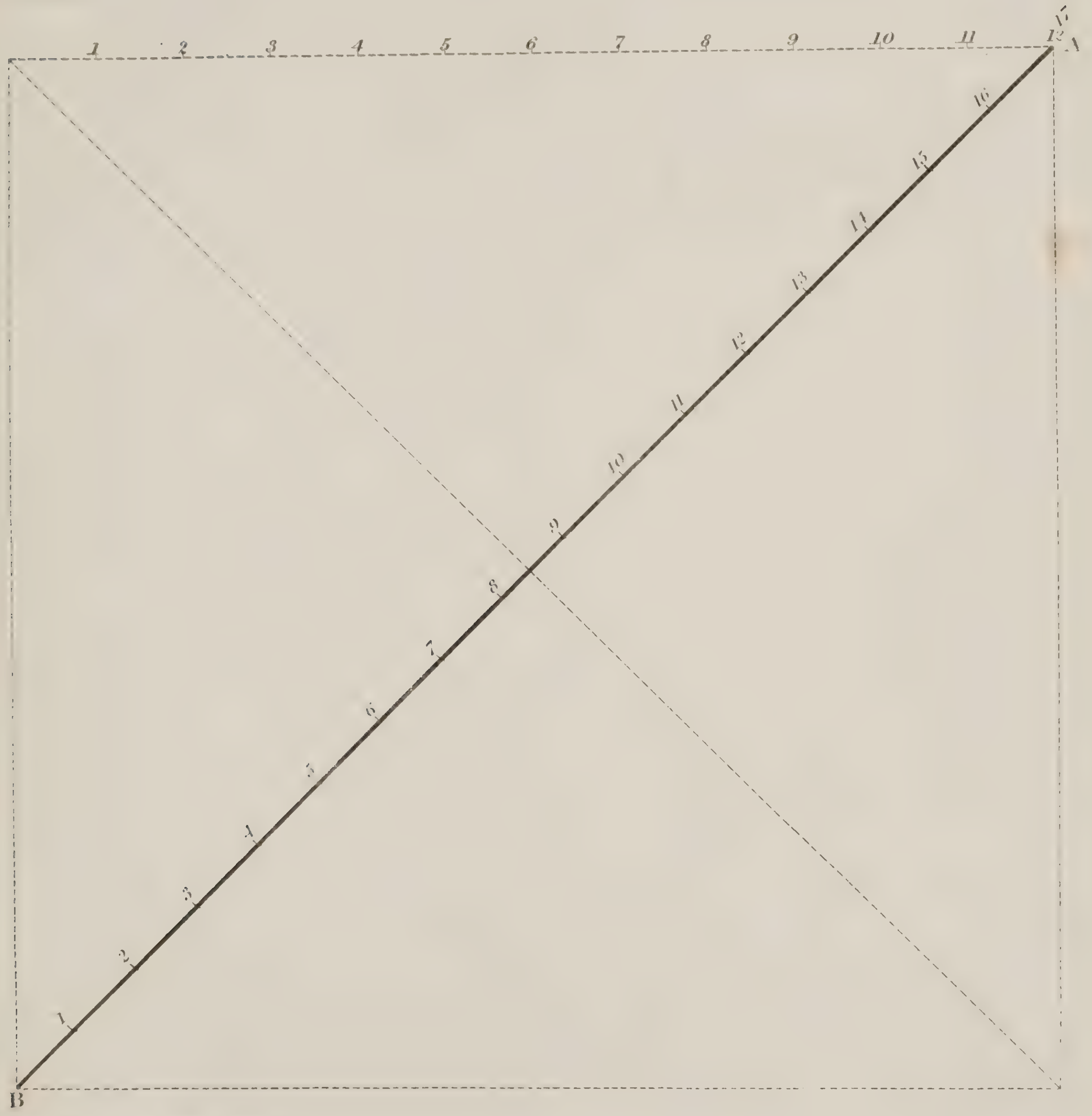


PLATE XII.

The hyperbola divided into various parts. This diagram may be, in some instances, of considerable utility, when any alteration is required for large door-ways or windows of buildings or churches. It will extend also to furnace-doors, &c., for steam-engine work, the circular head falling into the divisions with equal order and precision.

PLATE XIII.

From a given square produce a circle containing the same area. Draw the diagonal lines as shown, and divide one portion of the same into fifteen parts ; one point of the dividers placed at the centre, the other at twelve, describe a circle, which will be of the same capacity as the square. This diagram will always be of practical utility, and shows infinitely more than can be explained in any written description. In the square there are four triangles ; each of those triangles contains one-fourth of the square ; the quarter circle, whose figure differs materially from the other, holds a similar area notwithstanding : and it may be ascertained from this axiom that a quarter circle and triangle, formed of the dimension these are, their quantities must invariably be equal.

PLATE XIV.

The relative proportions of any square that may be produced from this figure may be accurately ascertained by the diagonal line AB, which in the true figure will be seventeen and the side twelve.

PLATE XV.

The division of the circle into aliquot parts. The direction of this figure follows in its nature precisely the same as the square, and the parts are equally explicit. In taking the circle herewith shown as the plan for the cylinder, the aliquot parts of the cylinder may be ascertained from this to any extent required.

PLATE XVI.

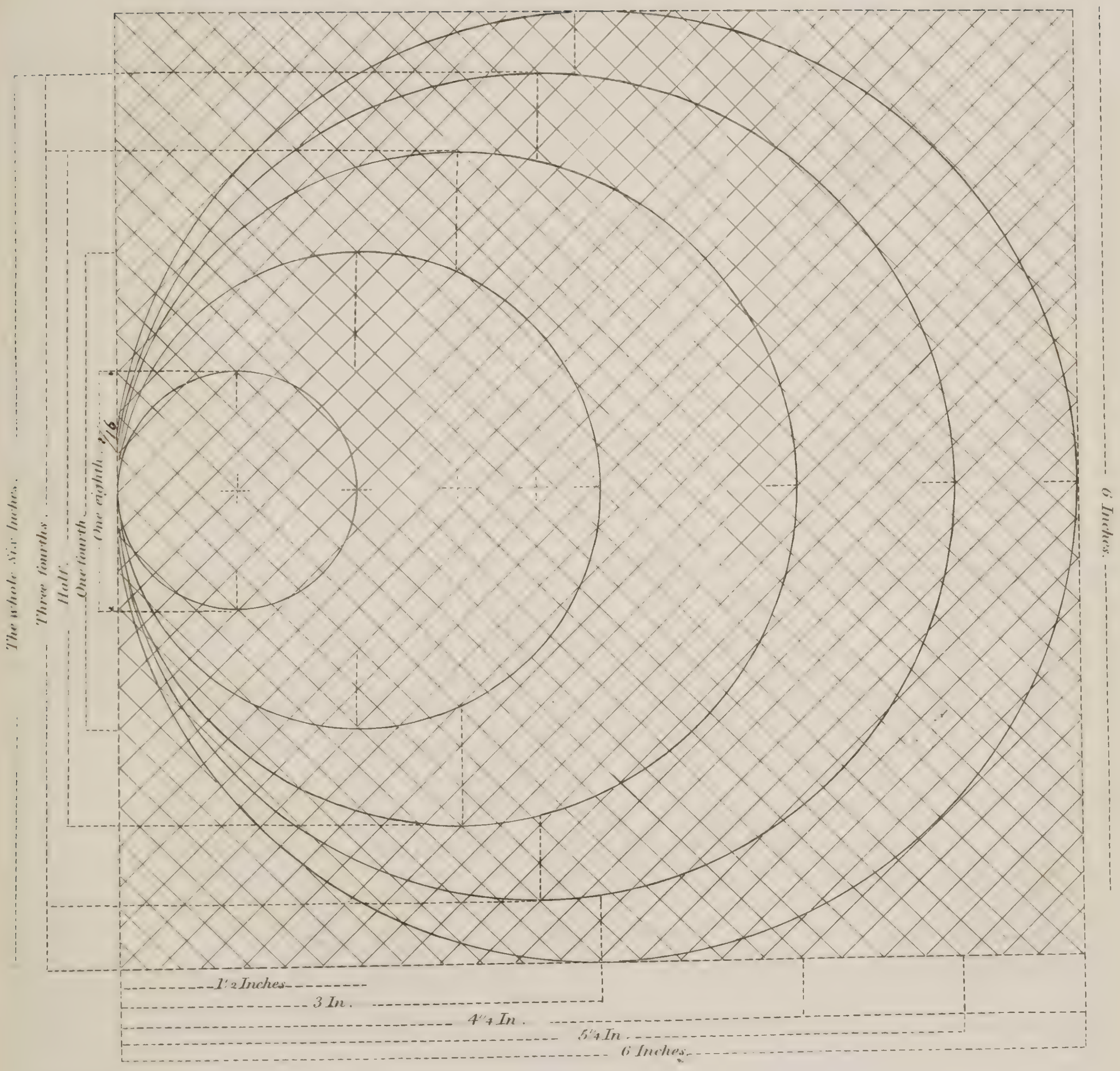
It is to be perceived, from this frame, that the external and internal A's have the same area or capacity as the middle part B. Therefore, if the quantity of B were to be deducted from A, it would be exactly half.

PLATE XVII.

The right-angled triangle given, to be made square, holding the same area or capacity. The triangle ABC is here introduced without reference to any dimension; therefore, let the diagonal line AB be divided into thirty-four equal parts, set off ten parts from A and ten from B, leaving fourteen in the middle; introduce the lines ED and DF, which will produce the square, as the diagram shows, holding the same space or area as the triangle.

CIRCLE.

FIG. 15



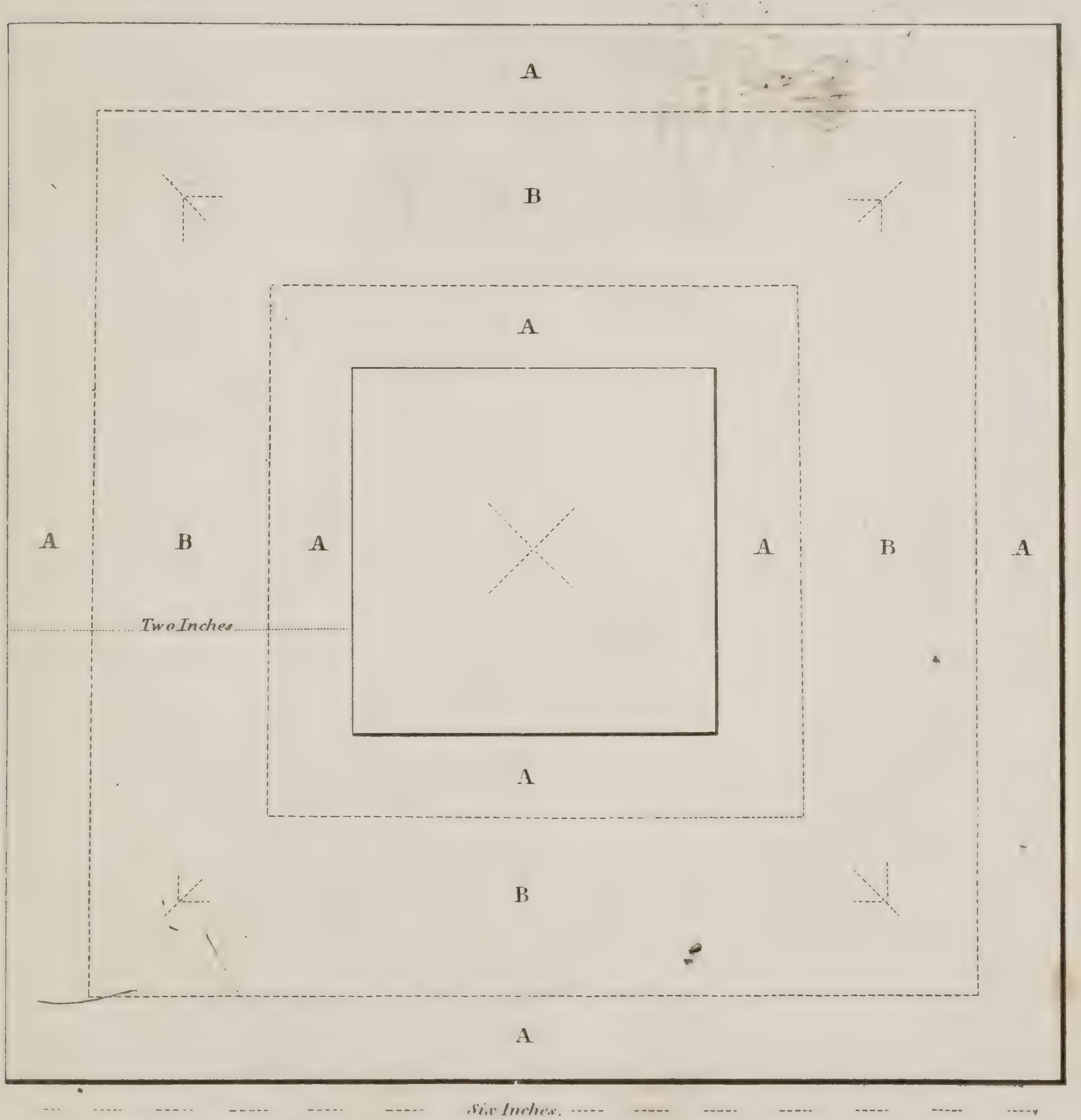
Invented and Drawn by John Bennett.

[illegible] nnett . 4. Three Tun Pass[illegible]

FRAME.

The dotted space B.B.B.B. is half the area of the frame A.A.A.A.

FIG.16.



Invented & Drawn by J. Bennett

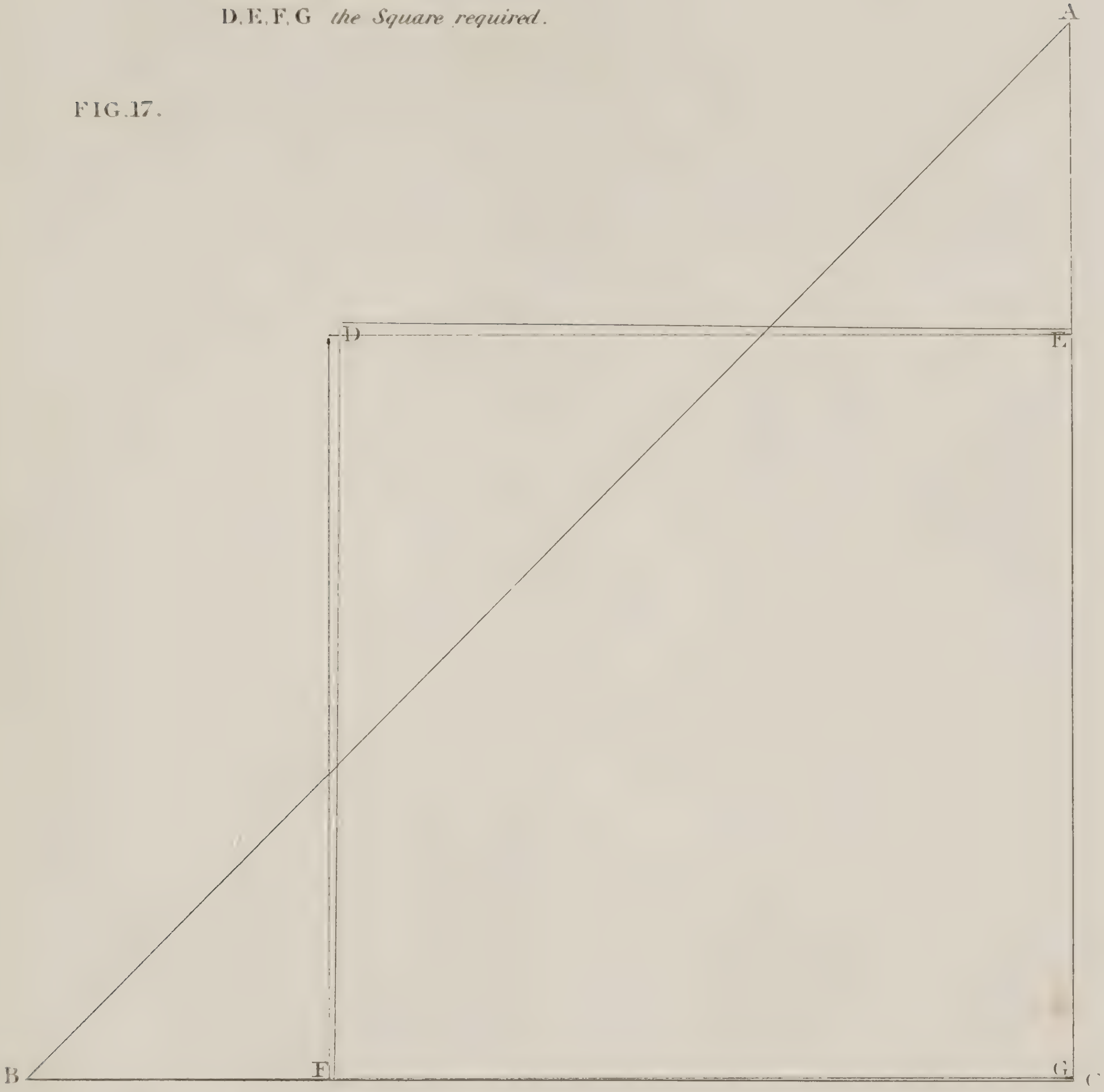
[illegible] Passage Ivy Lane 1836

TRIANGLE.

The Triangle A,B,C, *being given, a Square is required occupying the same Area only.*

D,E,F,G *the Square required.*

FIG.17.

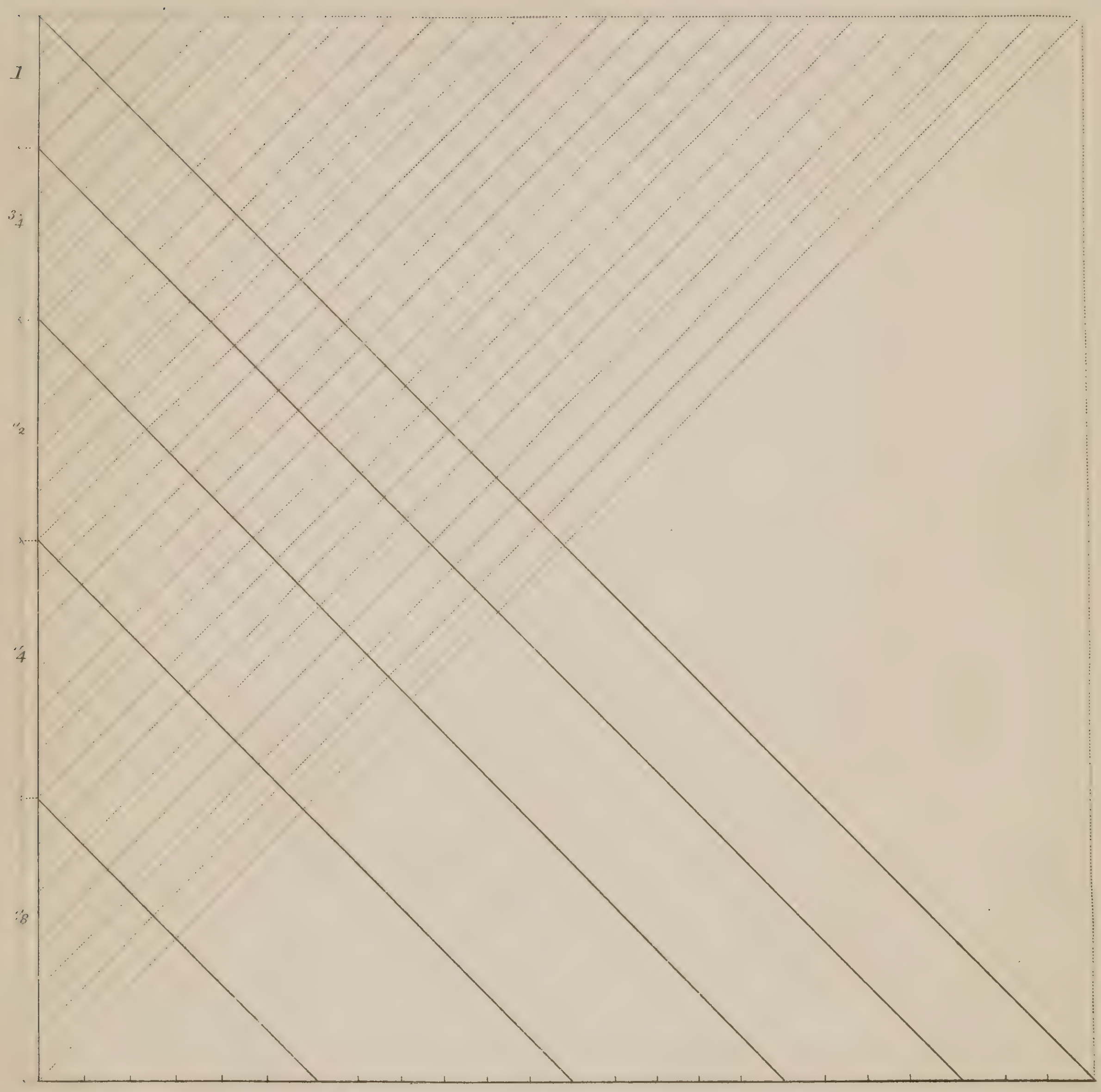


Invented and drawn by John Bennett

John Bennett 4,Three Tun Passage Ivy Lane 1836

RIGHT ANGLED TRIANGLE.

FIG.18.



Invented and drawn by John Bennett

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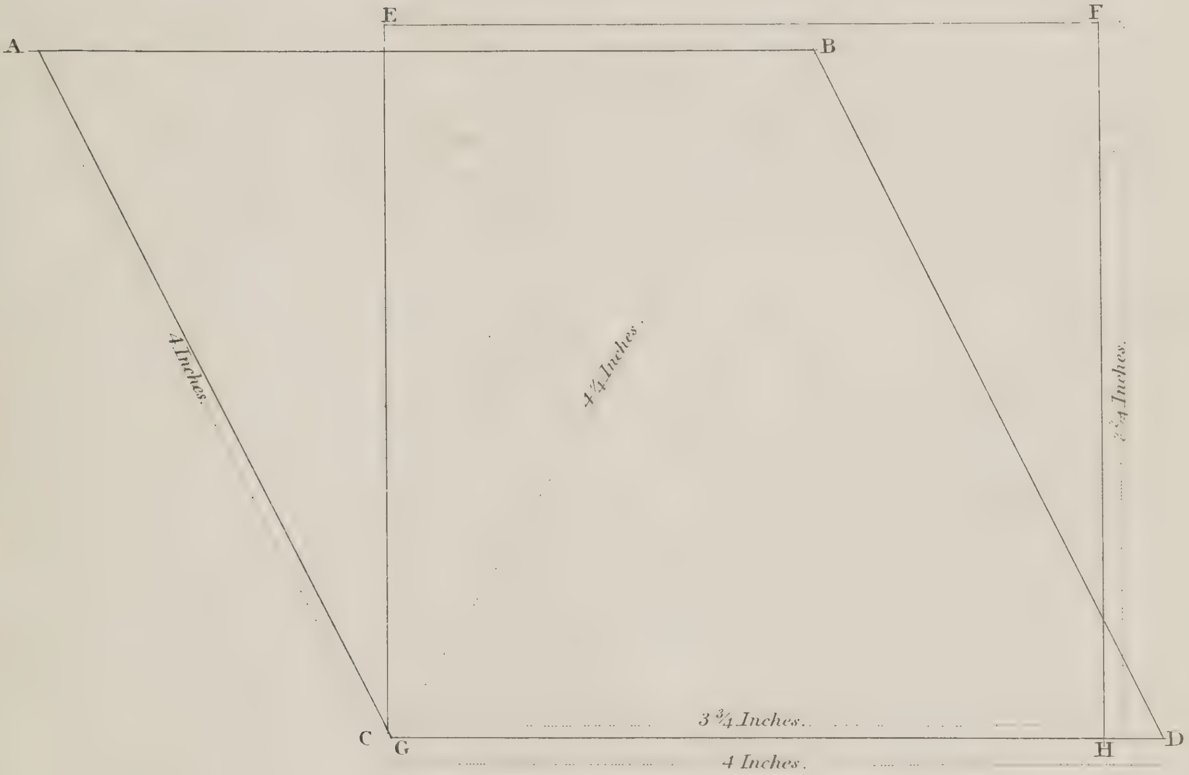
RHOMBOIDES.

A Rhomboides being given, as A, B, C, D; a Square the same Area, is described as E, F, G, H.

FIG. 19.



Invented and drawn by [illegible].

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PLATE XVIII.

This right-angled triangle being half of the square, is herewith introduced, divided by the dotted lines as shown, into aliquot parts: namely, three-fourths, half, one-fourth, and one-eighth parts, the division of which will be found set out upon the base of the square; viz., six for the eighth, twelve for the fourth, seventeen for the half, and twenty-one for the three-fourths parts; from this scale of divisions any intermediate quantity of this, or any other similar triangle, may with the greatest accuracy be obtained.

PLATE XIX.

The rhomboides ABCD given, whose dimensions are four inches by three and three quarters inches, is required to be made into a square, containing a similar area. Let a perpendicular fall from the vertex B upon the base line GD, which produces three inches and a half; let that be multiplied by the content of the line AC or CD, either of which are four, the result will be fourteen, the contents of the square EFGH. Or divide the side of the square at EG into twenty-eight parts, set off another part in addition for the square as shown by the line EF. Consequently, the square holding the same area as the rhomboides may be readily obtained, as this figure elucidates.

PLATE XX.

Shows the rhomboides divided into parts, viz., an eighth, fourth, half, and three-fourths: the divisions are not only correct in their respective areas, but take the precise form of the original figure in all its bearings. This diagram will serve to demonstrate the properties herein laid down, that notwithstanding the nature of the figure, whether true or otherwise, the divisions are brought out perfect.

PLATE XXI.

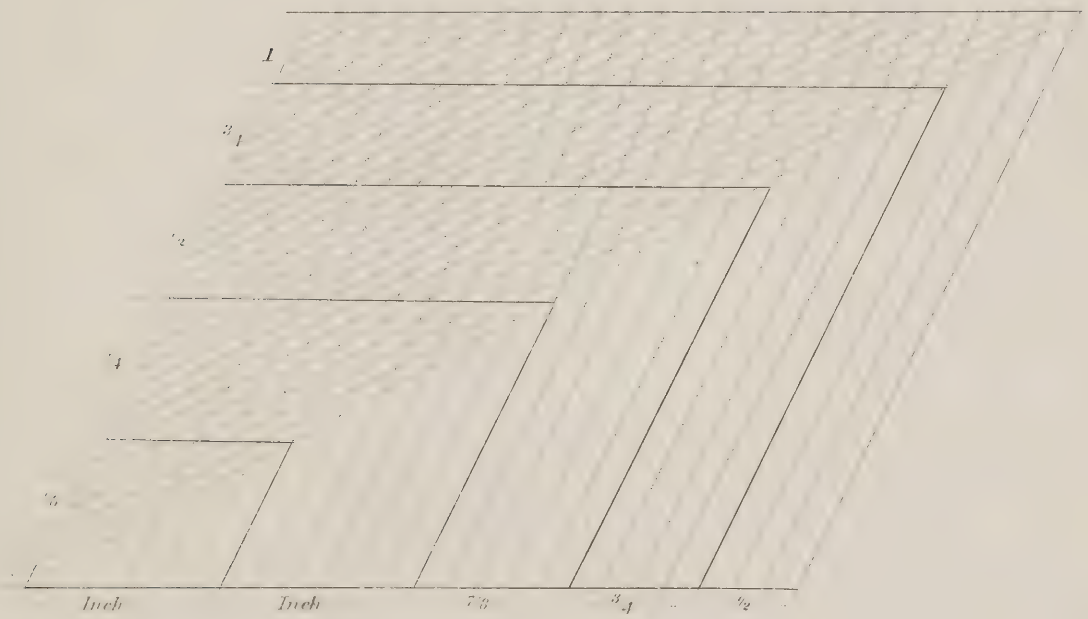
The equilateral triangle, divided by the same method as the figures before explained; and any fractional part of either of those divisions can be found by this scale.

PLATE XXII.

The divisions of this square are clearly exemplified in the preceding plates; but the introduction of this deviation may be extremely useful in taking any intermediate equal part from the whole; or of adding to, and deducting from, such parts of the figure as may at any time be required. In bordering or frame-work, and many other useful dispositions in workmanship, this diagram may be held in requisition, more especially where a correct attainment of the divisions is considered to be of any consequence.

RHOMBOIDES.

FIG. 20.



vented and drawn by John Bennett

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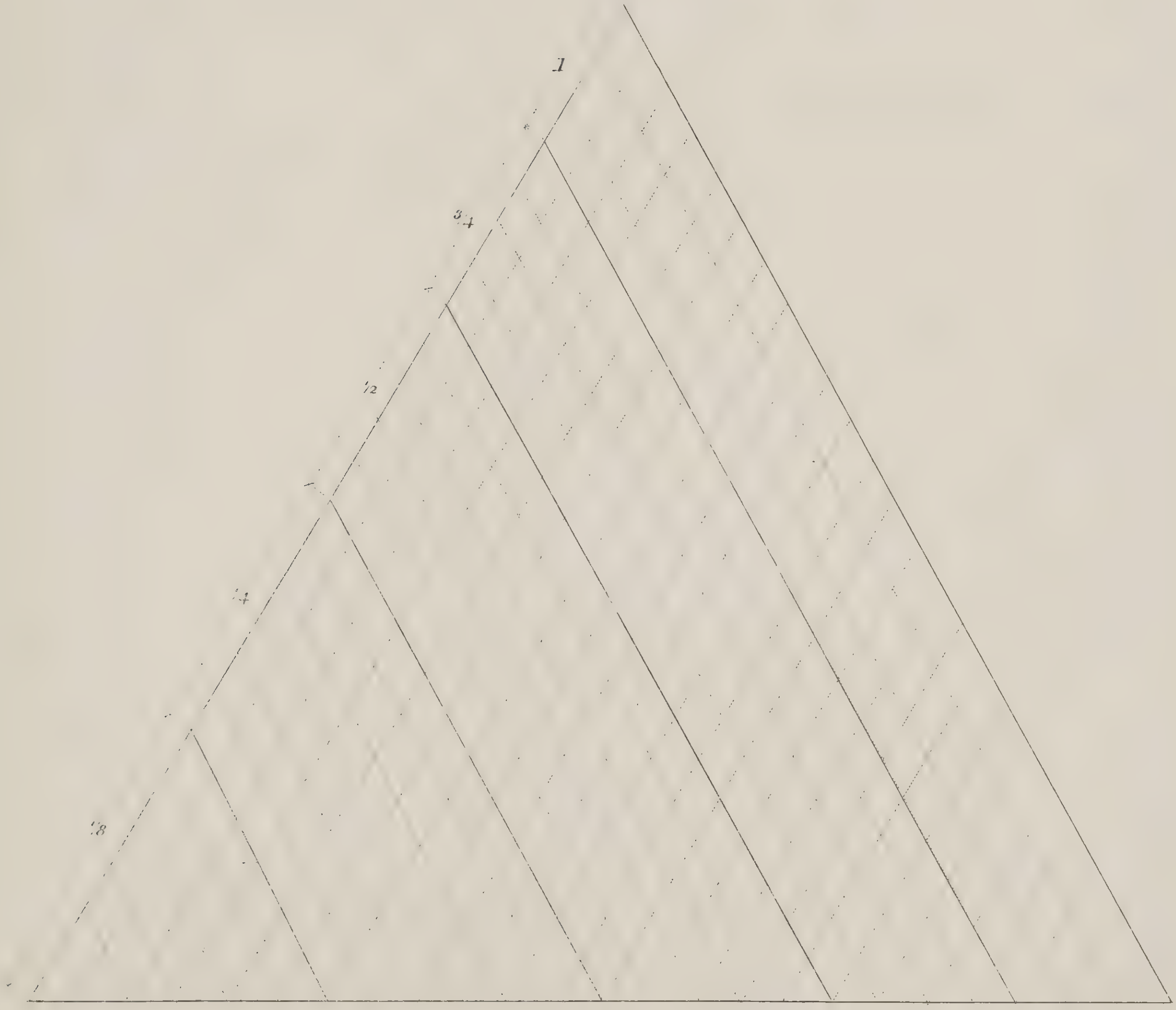
EQUILATERAL TRIANGLE.

FIG. 21.



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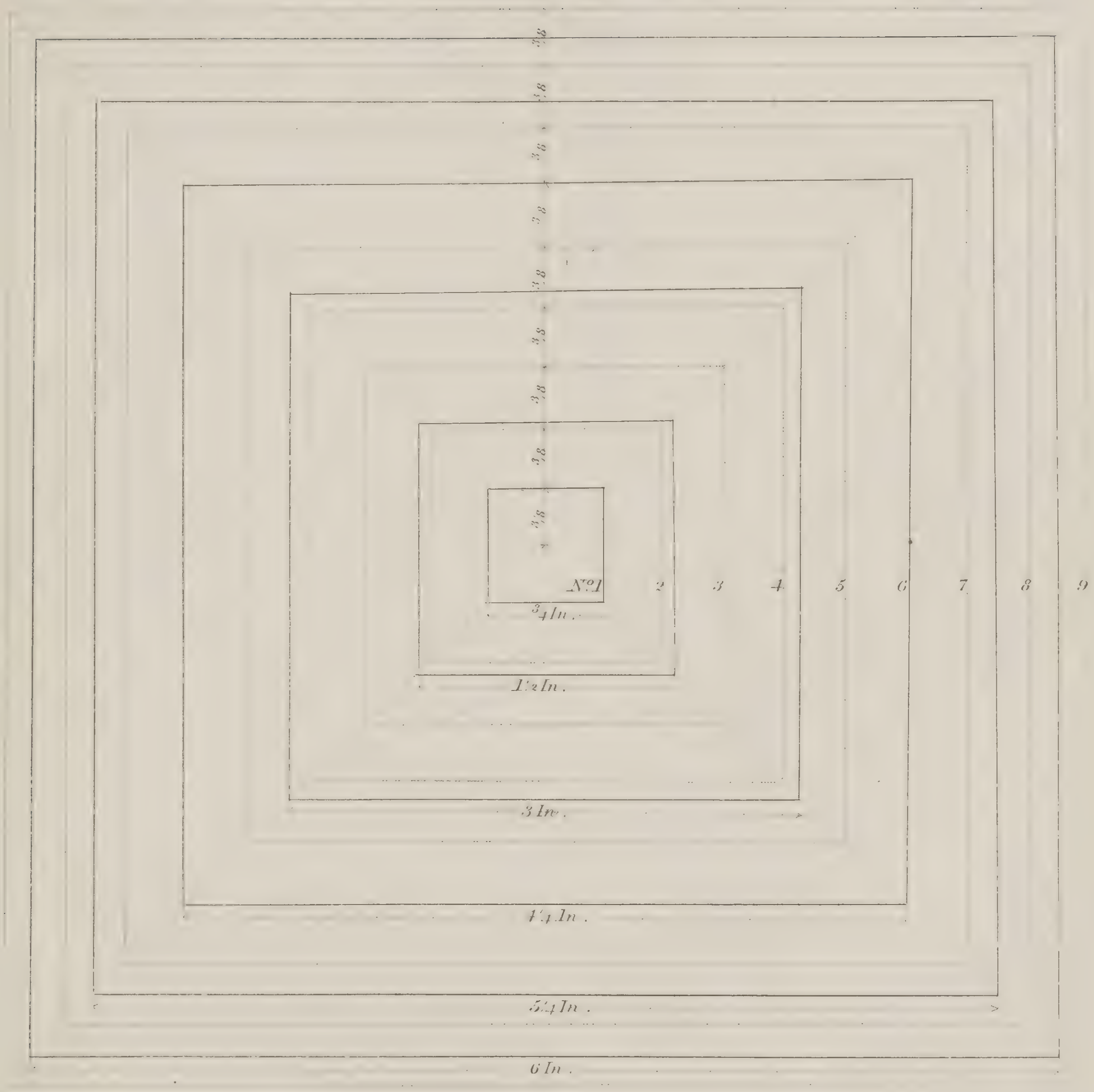
SQUARE.

FIG. 22.



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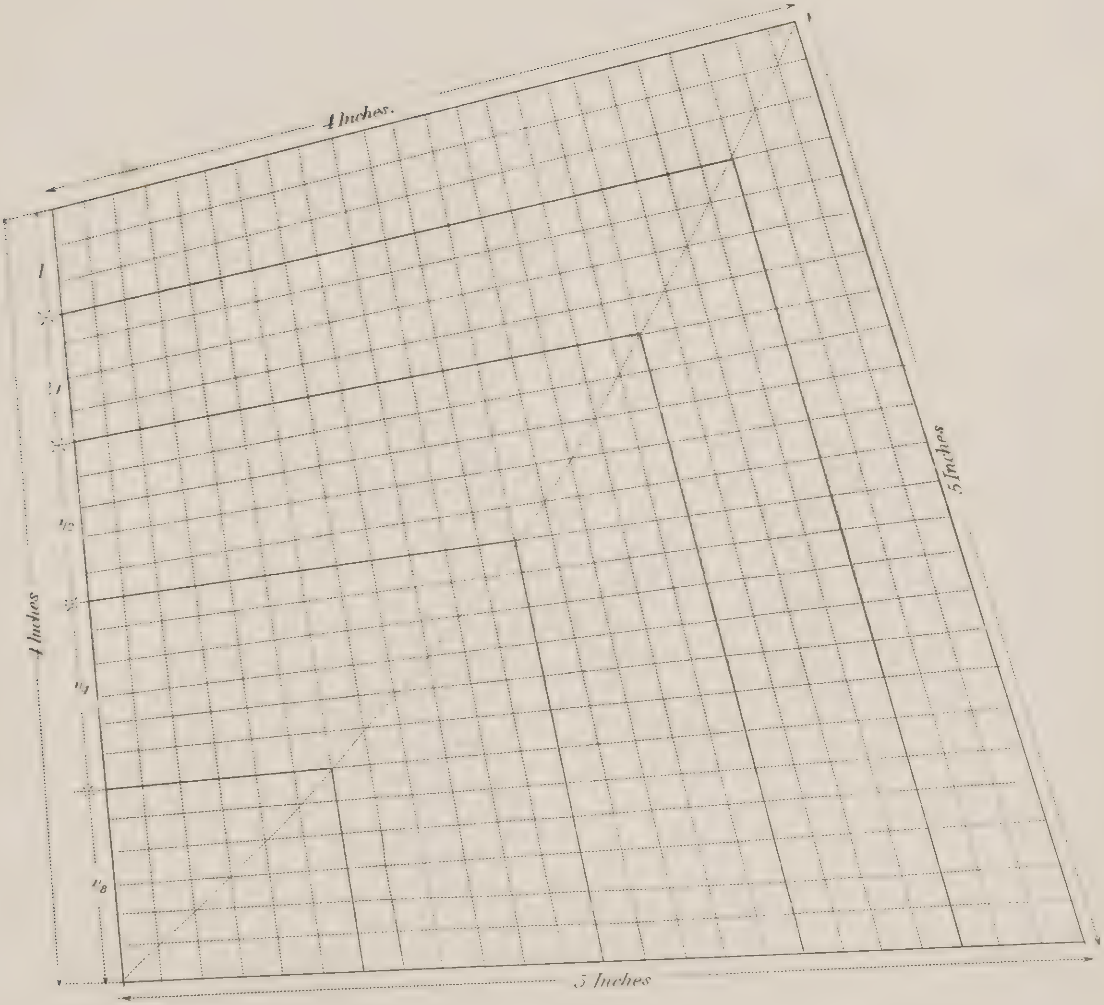
IRREGULAR.

FIG. 23.



Invented and drawn by John Bennett

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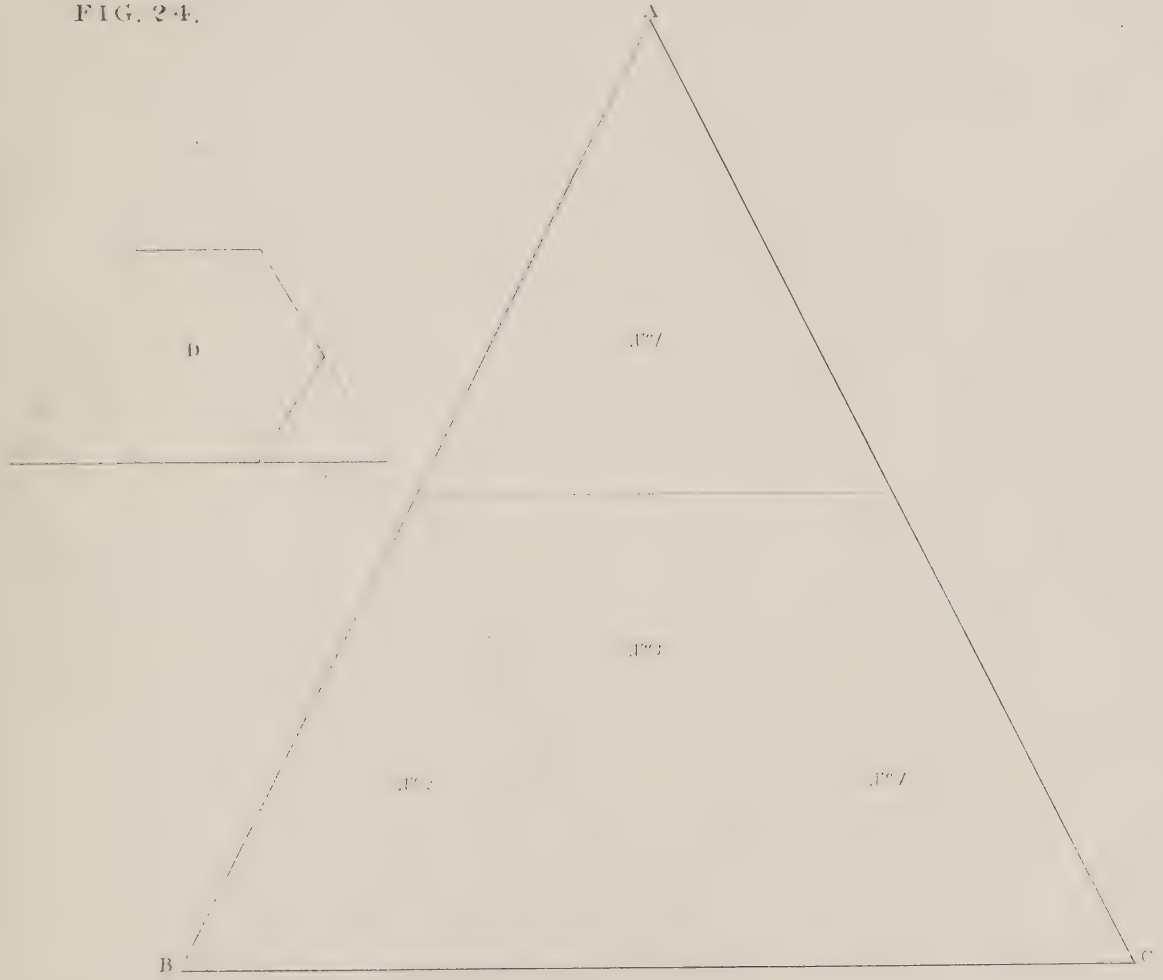
TRIANGLE AND HEXAGON.

The Triangle A.B. C. *being given ; let the same be divided into 4 equal Triangles. Bisect, the 3 sides, to which draw the lines, as shown in the following diagram.* — D. *the hexagon formed of the triangle.*

FIG. 24.



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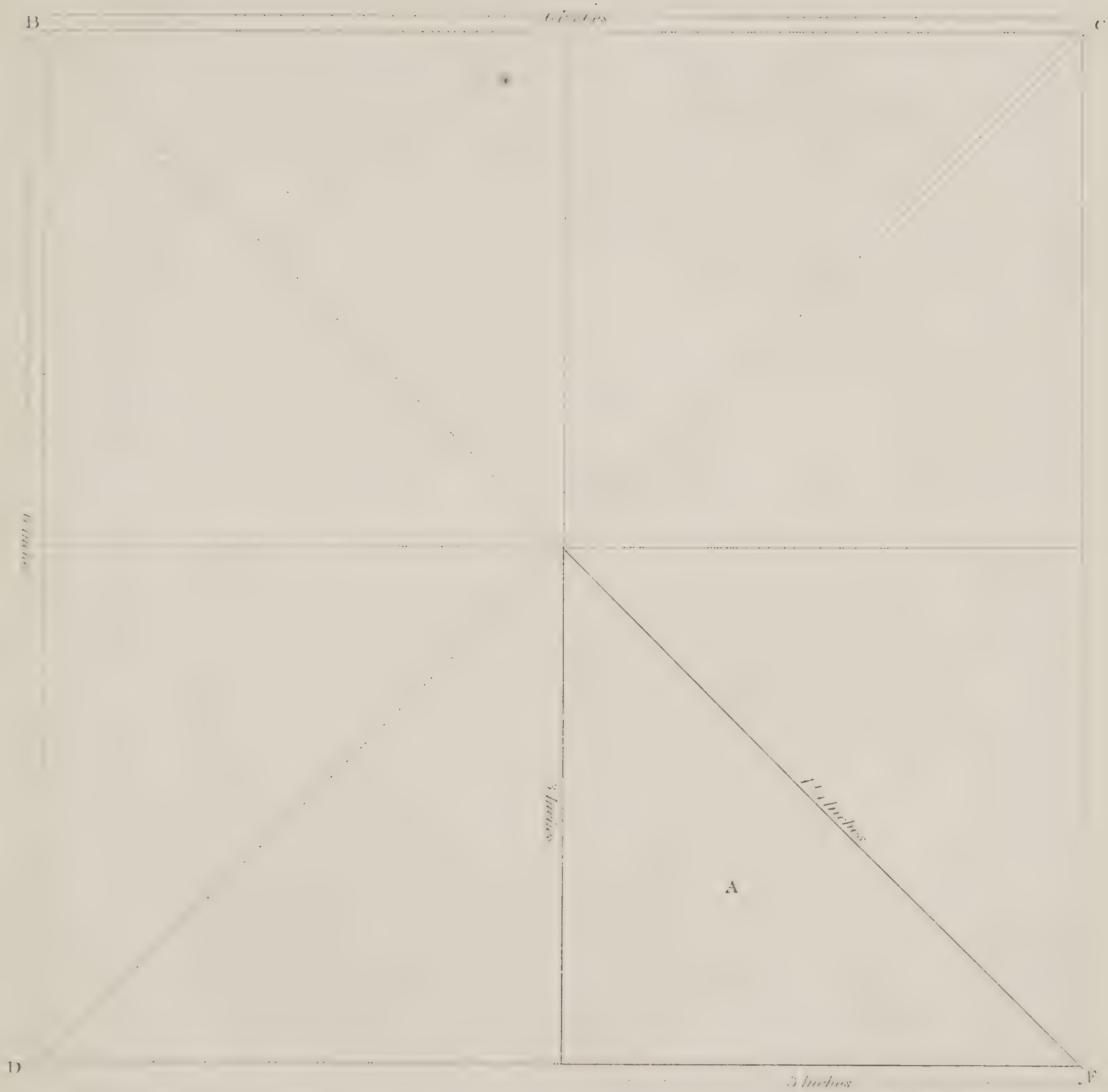
TRIANGLE
AND
SQUARE.

FIG. 25.



Invented and drawn by John Bennett.

The triangle **A**, *being given, and difficulty arising respecting its quantity, by forming the square* **B.C.D.E.** *to it; the contents may speedily be ascertained: for instance, the square being 6×6.=36: that divided by 8, leaves 4½; then 3×3.=9: half of which, is 4½, which was to be demonstrated.*

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PLATE XXIII.

This irregular figure, comprehended under two sides of like dimension, and two sides of equal, but larger dimensions; although such is described by this diagram, it is nevertheless divided as any of the foregoing figures, whose dimensions are equal. The diagonal line in this figure appears curved, which must be the case where the sides do not correspond; but, where the sides are equal, the diagonal invariably will be straight from point to point.

PLATE XXIV.

The triangle laid out into four divisions, the whole of which are equal in capacity and alike in figure: these divisions are found by bisecting the sides, and drawing the dotted lines as shown. This method will perform the like operation to any three-sided figure. The hexagon, or six-sided figure, as shown at D (providing the sides of the triangle are equal), may be obtained within it, by dividing the three sides into three equal parts, as is described.

PLATE XXV.

The square disposed or laid out into eight equal triangles. Half of this square, from B to D, forms the rectangle, and composes four of the triangles : the contents of one of those triangles, multiplied by eight, will introduce the area of the square, and by four, the area of the rectangle, being half the area of the square ; and the triangle CBD, being half the contents of the square, will also be equal in quantity of area to the rectangle.

PLATE XXVI.

The ellipsis, or oval, divided into equal parts, viz., an eighth, fourth, half, three-fourths, &c. Upon the face of this plate will be observed a reference to the rhombus, No. 2, page 7, which is incorrectly placed ; it should be to the rhombus, No. 1, Fig. 9, wherein the method which is adopted for the equal division of that figure will directly apply to this. By the dotted lines from A to B the points are struck circular from a centre, which may be found ; so that the lines should intersect each other without crippling. No. 1 constitutes the eighth division; No. 2, the fourth ; No. 3, the half ; and No. 4, the three-fourths part of this ellipsis ; and by this method any division of the ellipsis may be obtained, without reference to the transverse, or conjugate.

PLATE XXVII.

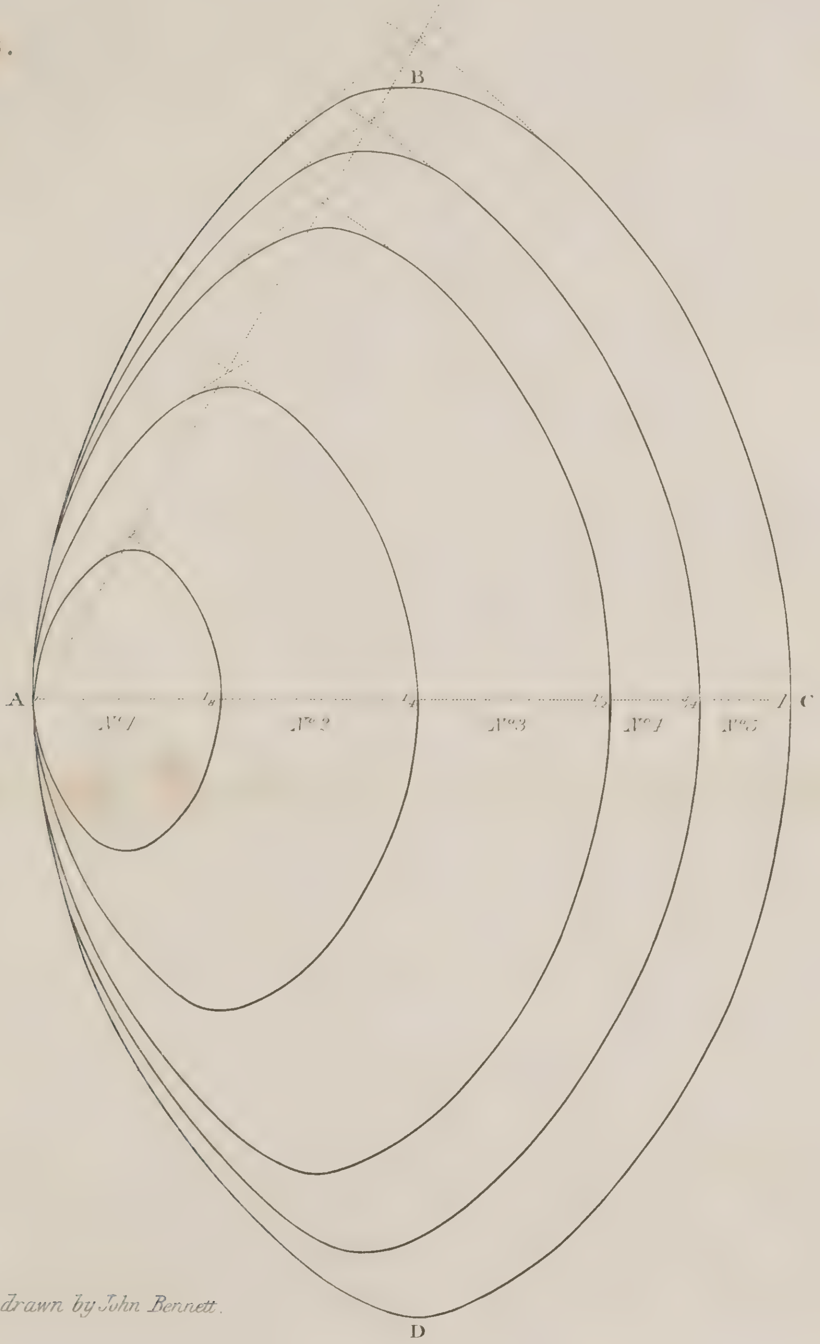
The section of an irregular pyramid, or four-sided figure, whose sides are equal, being six inches, as the plate demonstrates ; notwithstanding such inequality, it is divided into aliquot parts : viz., one-eighth, fourth,

ELLIPSIS.

An ellipsis being given, as A, B, C, D, *let the same be divided into parts, viz.t an eighth, fourth, half, three fourths, &c. as No.s 1, 2, 3, 4, 5.*

See Rhombus No. 2, Page 7.

FIG. 26.



Designed and drawn by John Bennett.

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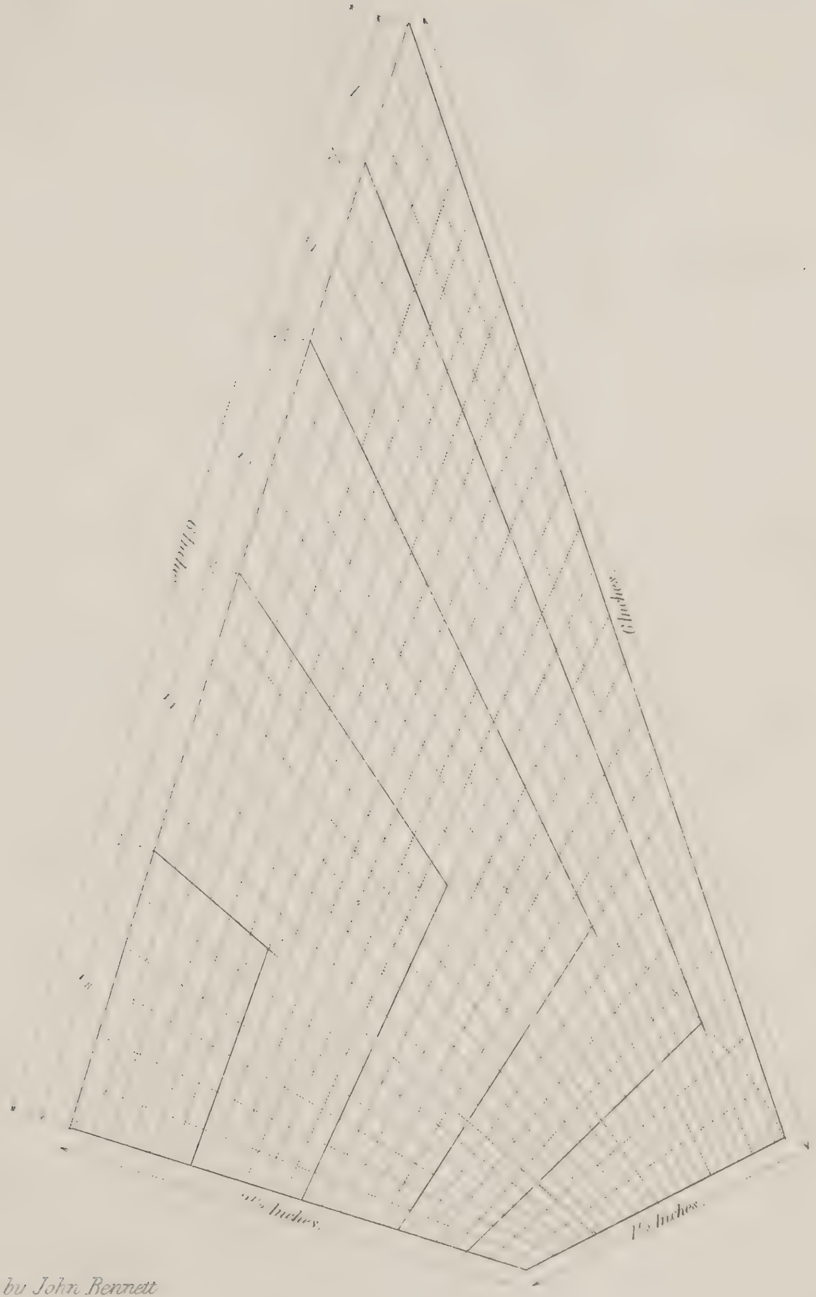
PYRAMID.

FIG. 27.



Invented and drawn by John Bennett

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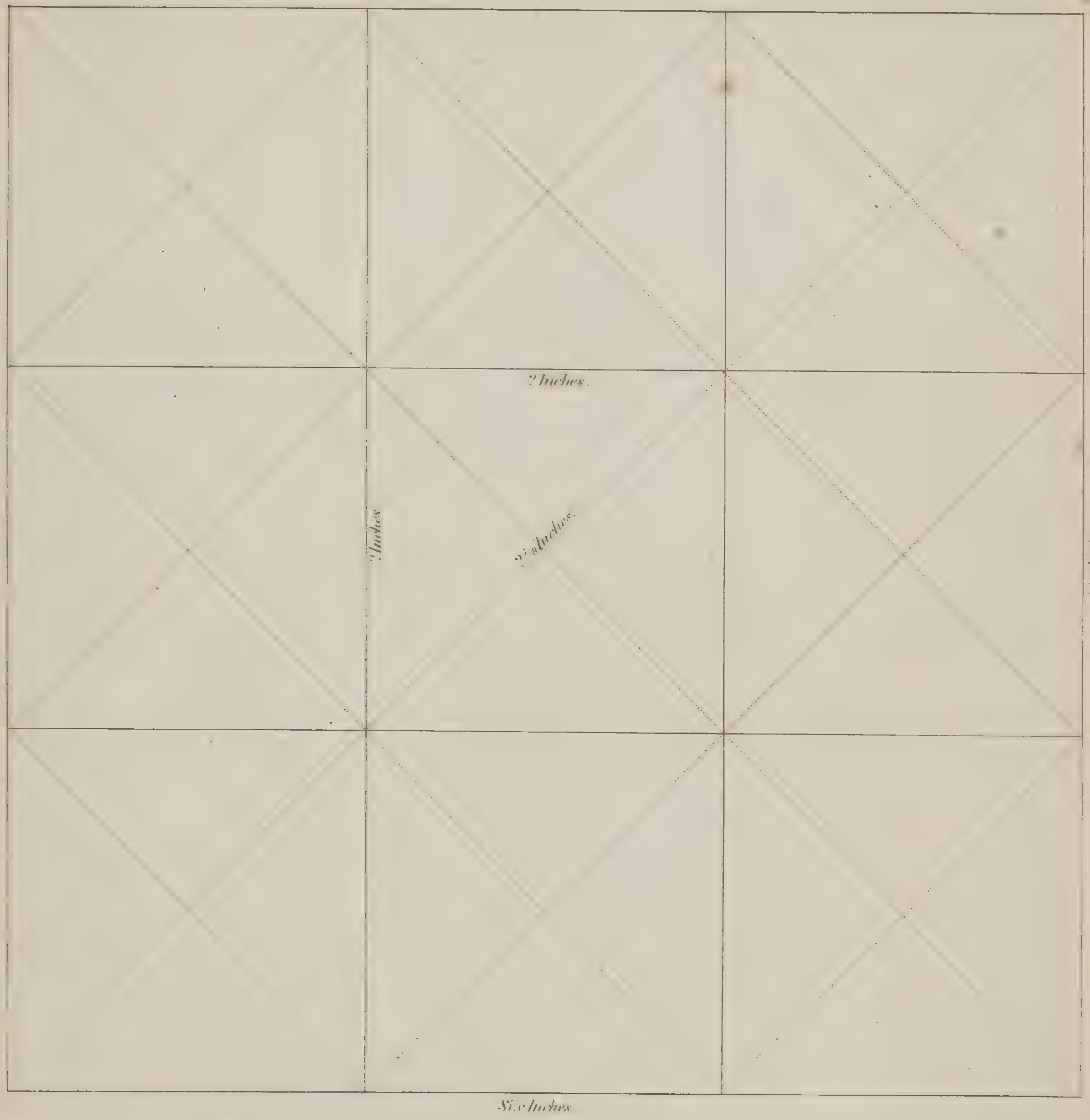
SQUARE AND TRIANGLE.

This Square is shown divided into 9 smaller squares, and 36 triangles; therefore, one of the triangles, must be a 36th part of the whole square: consequently, all the squares, and triangles, bear equal proportions to each other.

FIG. 28.



Invented and drawn by John Bennett.

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half, and three-fourths ; the areas of each respectively will be found, when worked out, perfectly correct ; although, in the representation of this figure, the smaller divisions bear but an indifferent resemblance of the original.

PLATE XXVIII.

This figure is shown divided into nine small squares, and thirty-six triangles ; therefore, one of the triangles must contain an area of the thirty-sixth part of the whole square ; consequently all the squares and triangles bear equal proportions to each other. In this figure much may be discovered relative to triangles, rectangles, and squares : for, by taking half this square diagonally, it will be found to contain eighteen small triangles, as are shown dotted upon the plate ; the base line of each, or either, will be two inches ; nine triangles, whose areas double the foregoing, the base line of which will be two and seven-eighths inches. The next larger has a base of three inches and seven-eighths, and contains nine of the small triangles, and doubles also the area of the last. And the triangle which holds a fourth part of the whole square, contains nine of the small triangles. Two-thirds of this square will form the rectangle, containing twenty-four small triangles ; and the remaining rectangle, whose dimension will be six inches by two inches, will contain twelve triangles : therefore, from the transformations this figure undergoes, much may be obtained in some cases in the reduction of a figure by triangles, in order to obtain some particular result or definition : for, in this it is made evident, if one triangle will not come in with even numbers, the next in succession may, and therefore by such means a correct solution may be obtained with infinitely less trouble than by any other.

PLATE XXIX.

The unequal four-sided figure, whose sides are respectively five and a quarter inches, four and three quarters inches, three and a quarter inches, and two and three quarters inches, is given in order that it may be made into a square, of the same capacity, or to contain the same area. The contents of the sides of the untrue figure, added together, as shown upon the plate, make sixteen, which, being divided by four, its number of sides, leaves four inches for the side of the square holding a similar area. In this instance, and probably in many others, this operation may be performed with accuracy; but it will not in all; therefore, should this method fail in producing a satisfactory result, others must be resorted to.

PLATE XXX.

The quadrangle, or rectangle, divided into equal or aliquot parts, viz., one-eighth, one-fourth, half, and three-fourths; in the formation of these divisions, it is not of the least consequence what dimensions the rectangle may be composed of: for the disposition by which this rectangle is laid out upon the plate, is certain to perfect these, or any quantity required, to the greatest possible nicety.

PLATE XXXI.

The parallelogram ABCD being given, a square is required, containing its contents, as EFGH, the square of the same area. From this

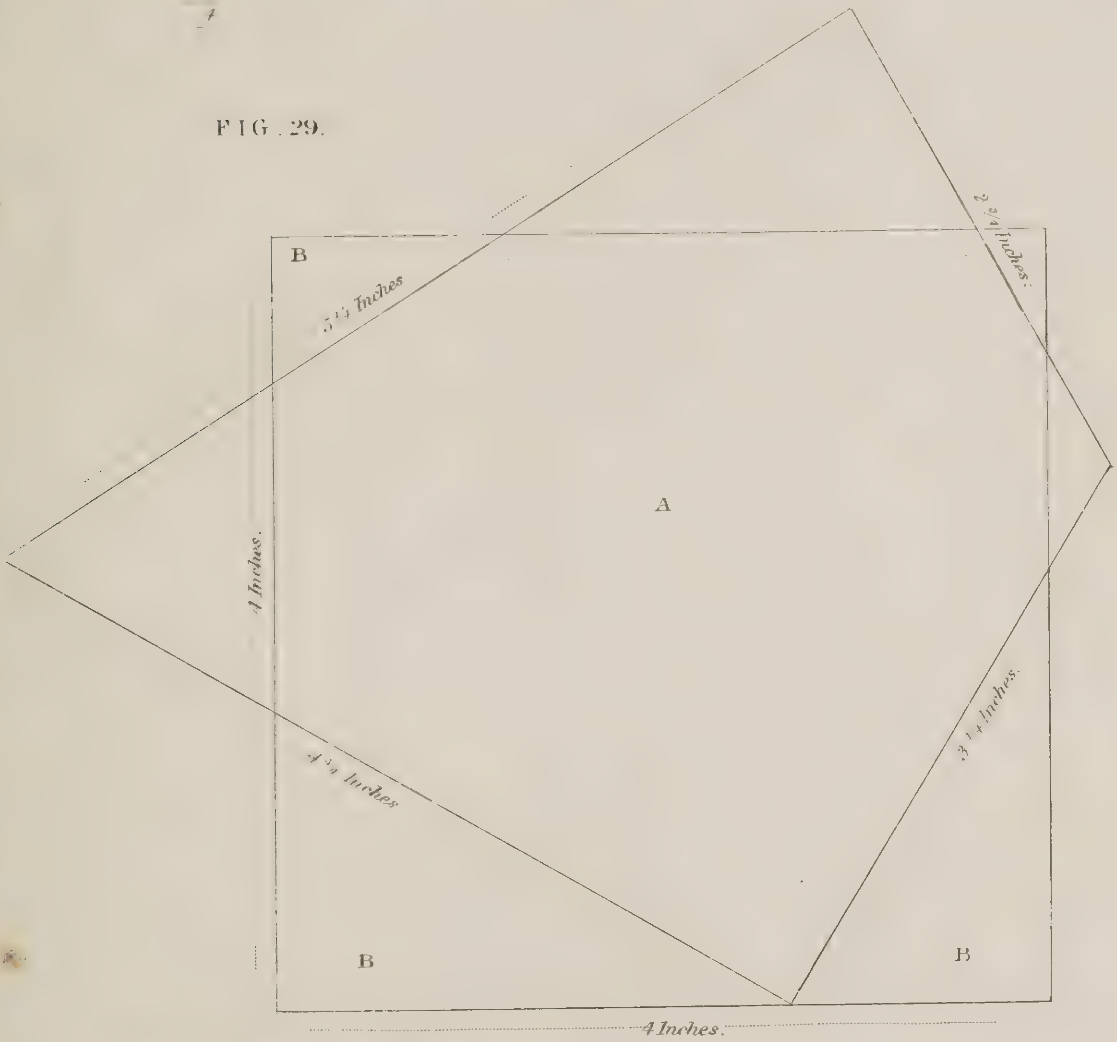
THE SQUARE, Nº 4.

Qu. *The figure A whose sides are all unequal viz.' 5¼ 2¾ 3¼ and 4¾ inches. it is required that an equal sided figure should be made, occupying the same area or space only?* area of A = 14.93

Ans.r B *the equal sided figure whose side is four inches.*; no. area of B = 16.

5¼
2¾
3¼
4¾
4/16
4

FIG. 29.



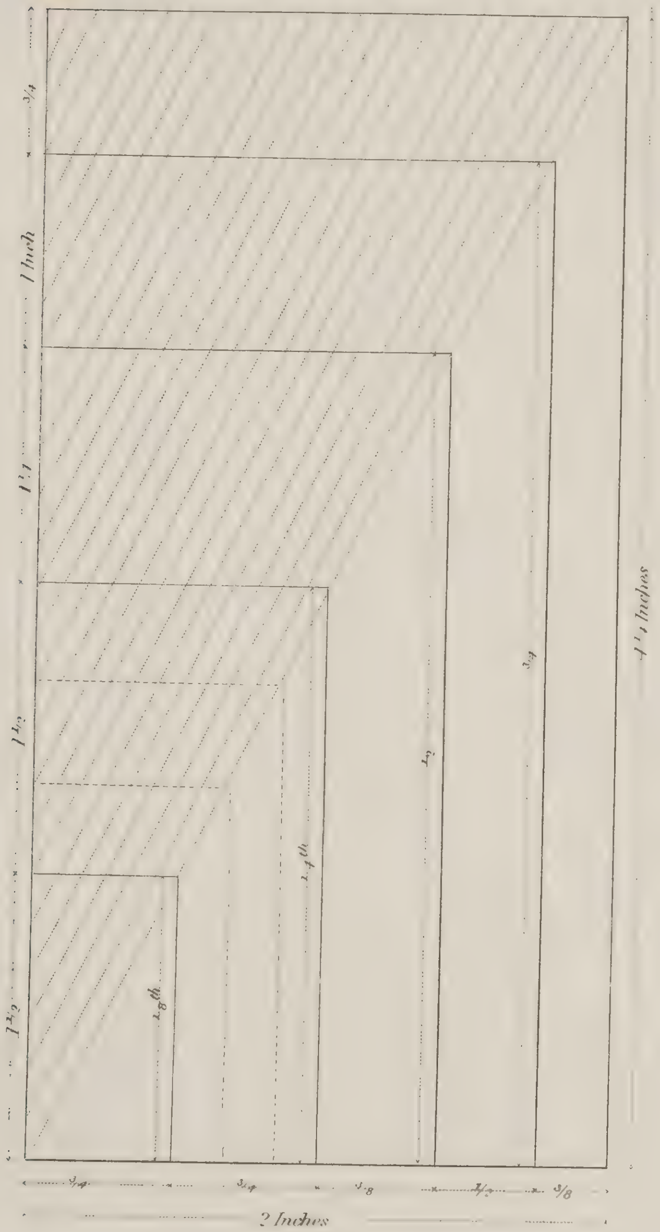
Invented and drawn by John Bennett.

London: John Bennett 4 Three Tun Passage

THE QUADRANGLE.

Divided into equal parts viz^t 3/8 1/4. 1/2. 3/4. and one inch

FIG. 30.



Invented and drawn by John Bennett.

London John Bennett 4 Three Tun Passage, Ivy Lane 1837.

PARALLELOGRAM.

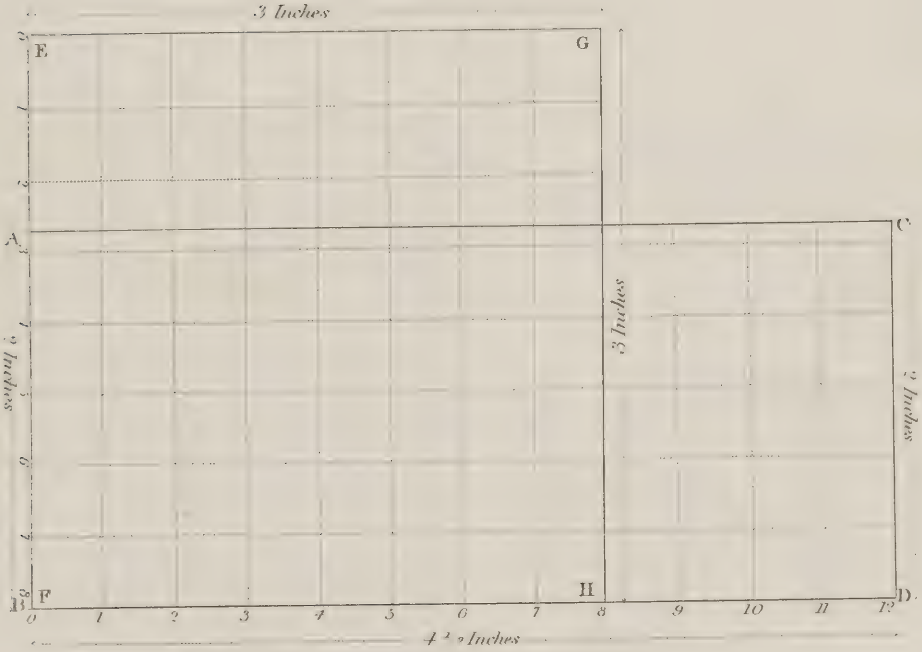
Q *The Parallelogram* A,B,C,D *being given, a Square is required, containing the same area only?*

A,E,F,G,H

FIG. 31.

Parallelogram. *4½ · 2 - 9 -*

Square. *3 · 3 - 9 -*



Invented and drawn by John Bennett.

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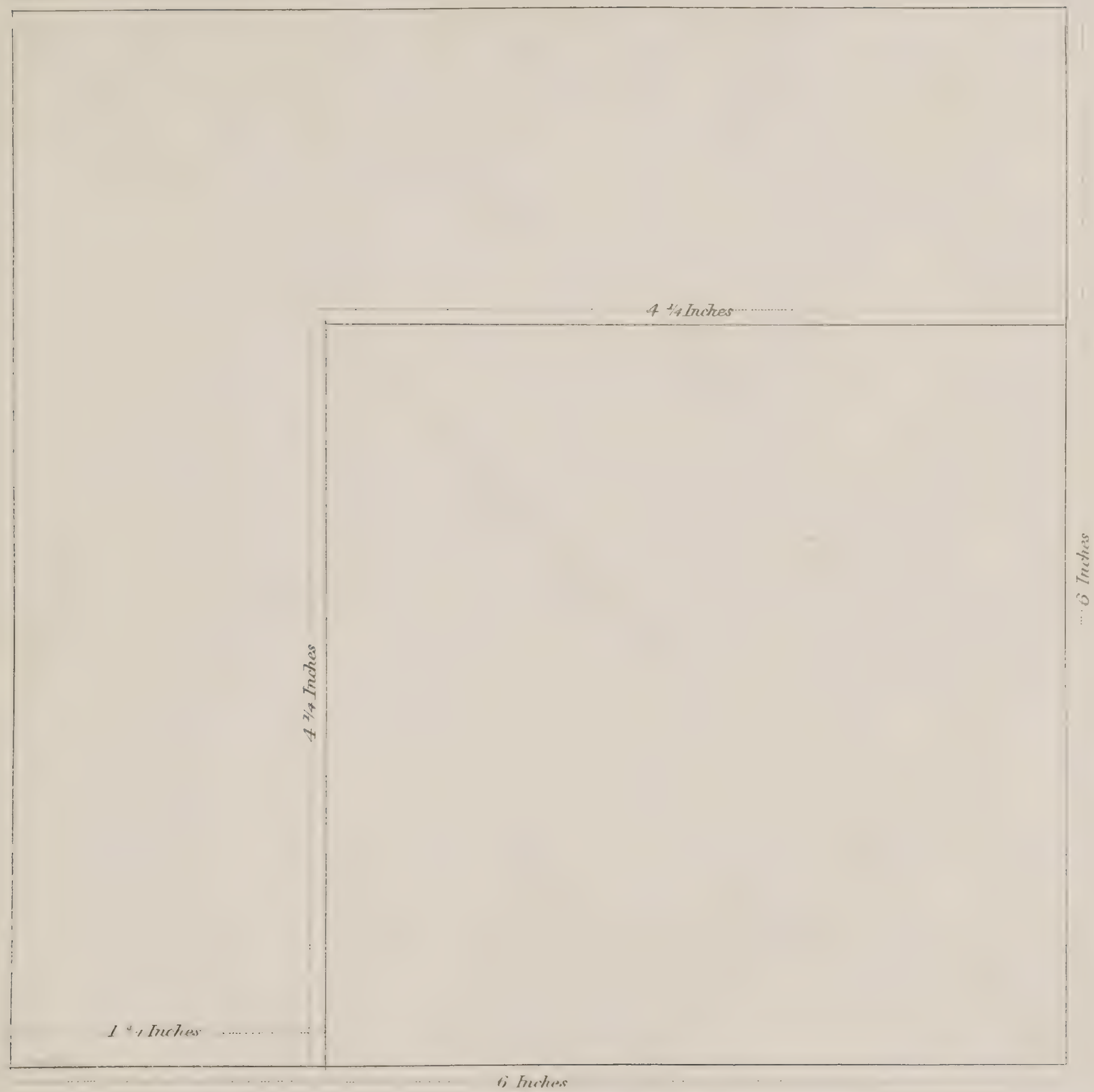
THE SQUARE, Nº 3.

Another method see Fig. 5.

FIG. 32.



Invented and drawn by John Bennett

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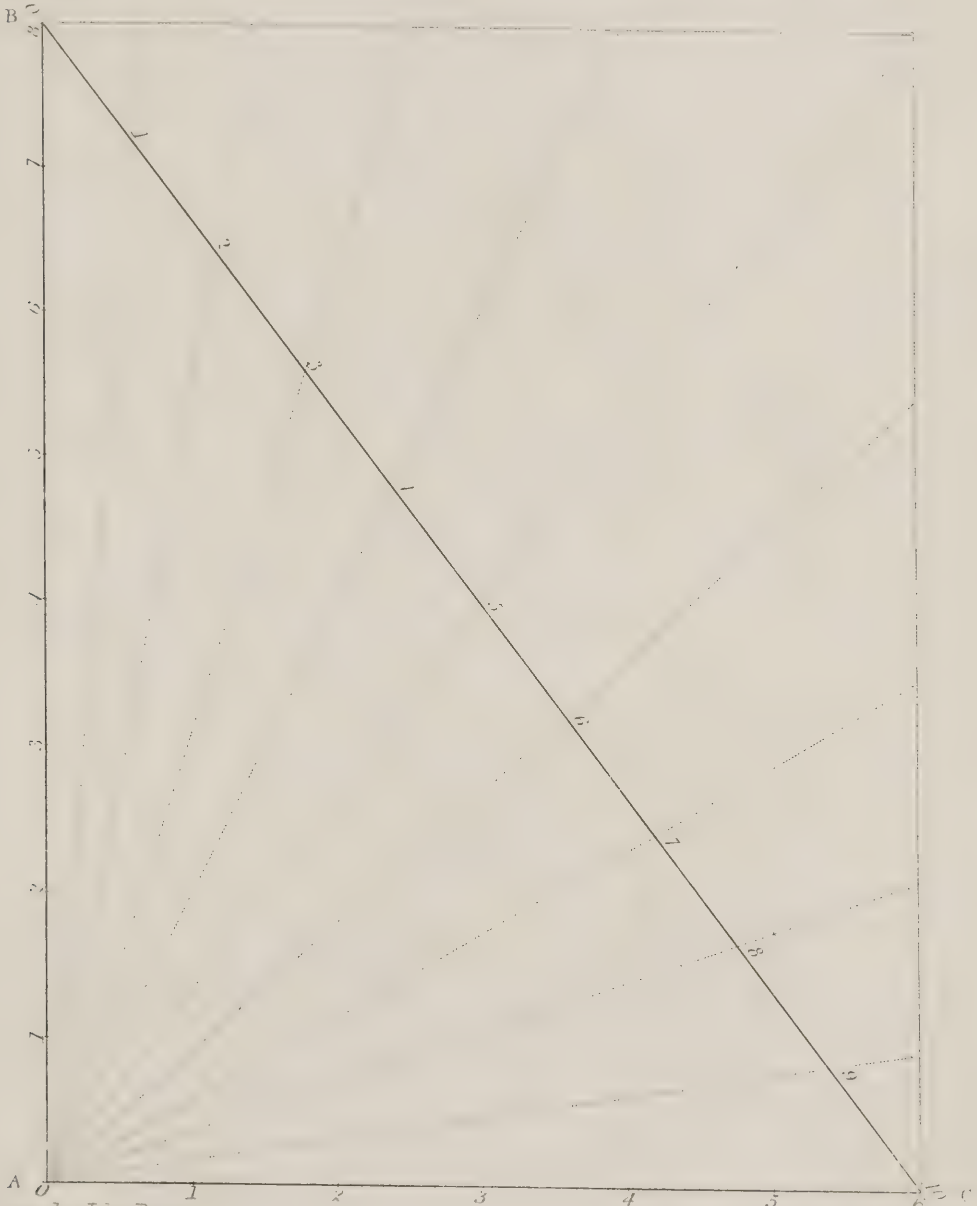
THE RIGHT ANGLE.

The Foundation angle of Six, Eight, and Ten, A.B.C. *making the fourth of the square.*

FIG. 33



Drawn by John Bennett

Scale ¼ of an Inch to one foot.

problem it appears to be an easy matter to square the rectangle, or parallelogram; and it is so whenever the quantities happen to fall into equal squares, as this figure demonstrates ; but, although it will sometimes and with some quantities work correct, it will be found by no means sufficiently so for general adoption.

PLATE XXXII.

In this figure the square of six inches is halved in its area by the square of four and a quarter inches, as is here represented. The halving of the square may be seen in a preceding figure, but in this it is to be further explained, viz., The dotted diagonal shows the square divided into two triangles, a horizontal line would divide the square in the middle into two rectangles ; therefore this square of four and a quarter inches would be equal in area to either of them.

PLATE XXXIII.

The triangle ABC shows the method by which a square may be obtained to the greatest truth upon any undetermined space. Let the line of six inches be made as this diagram shows, and also the vertical line of eight, and adjust the points of those to the diagonal of ten, as this plate serves to represent ; the angle at A will be right, and true for any workmanship to proceed from whatever.

PLATE XXXIV.

The equilateral triangle being given, let the same be made into a square, containing a similar area. Divide the side A into twenty-four equal parts, let them be drawn parallel and ranging with the opposite line, intersecting each of them upon the base line as shown. At part four, introduce the vertical line B, intersecting one side of the triangle in the eighth part; and as the base line shows from figure four to twelve, the distance of the square, let the rest be set out for the remainder of it, as this diagram serves to elucidate.

PLATE XXXV.

The crescent divided into eight sections, the pyramid is also divided in like manner; but the difference in their areas is as five to four, the area of the crescent being two and a half, and the pyramid two; it is therefore evident, from this diagram, that the curved lines of the crescent gain upon the straight lines of the pyramid.

PLATE XXXVI.

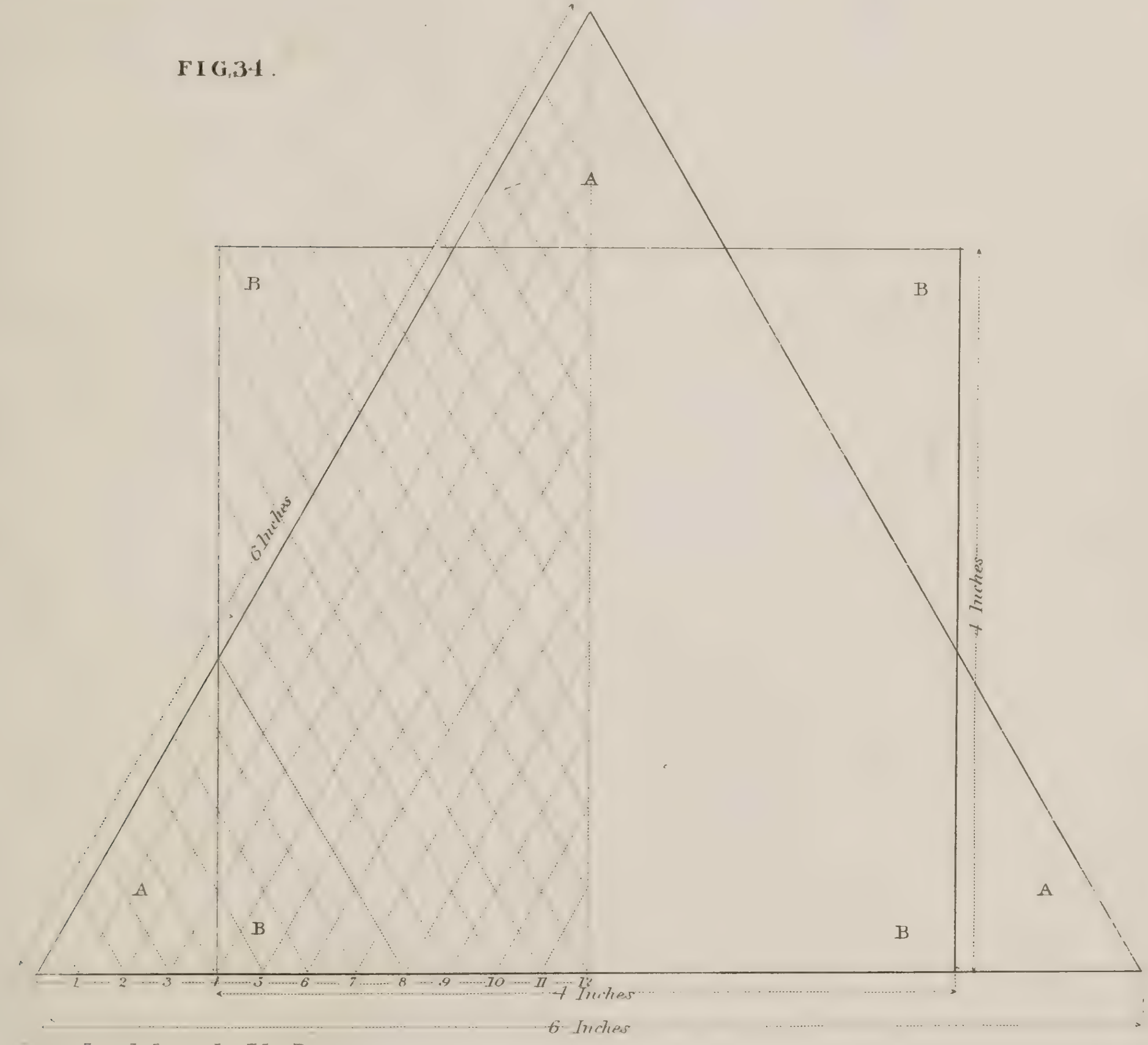
The pyramid divided into aliquot parts, namely, an eighth, fourth, half, and three-fourths; by the divisional lines as shown, the manner in which this is performed is similar to the preceding figures.

EQUILITERAL TRIANGLE
AND
SQUARE.

AAA.–Qu: *An equiliteral triangle being given, whose side is Six inches; required the side of a square, containing the same area?*

BBBB. *Ans.^r Four Inches.*

FIG.34.



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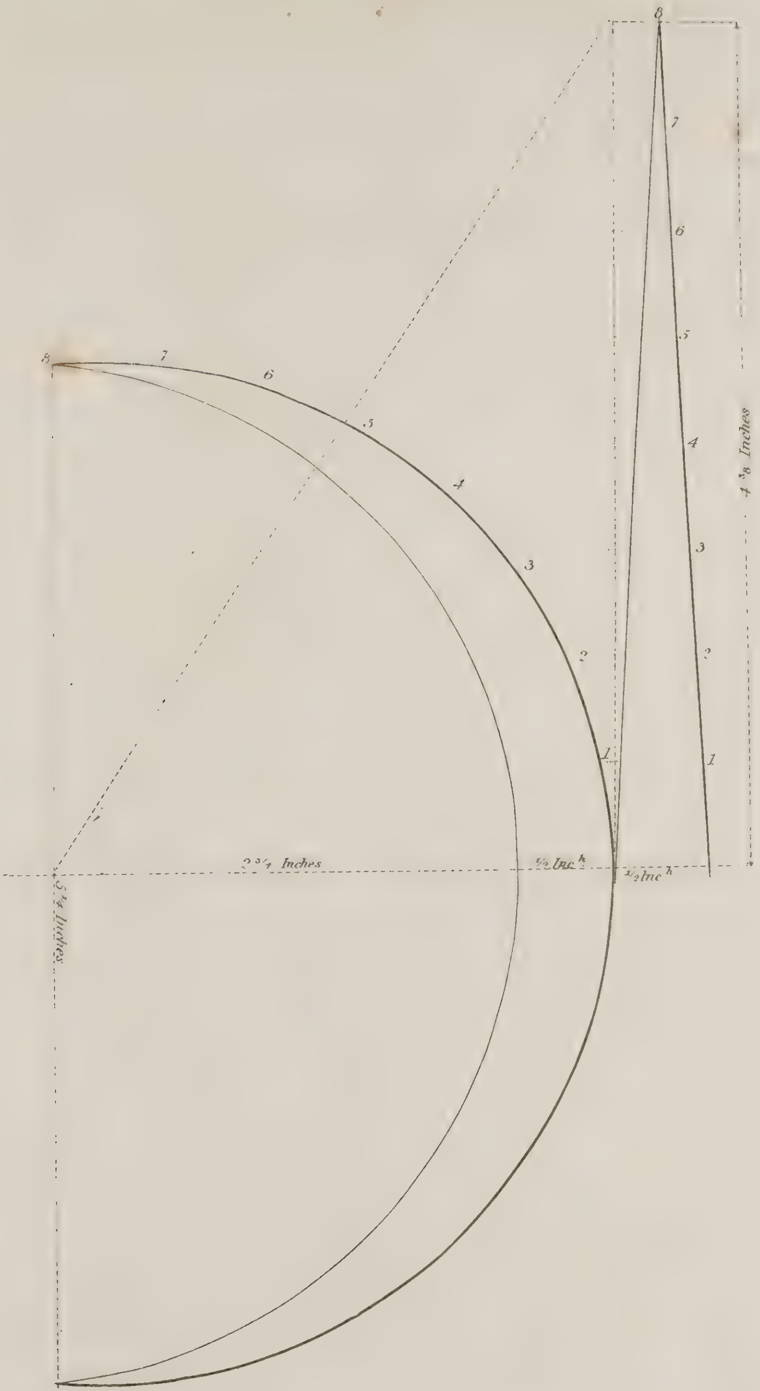
CRESCENT.

FIG. 35.



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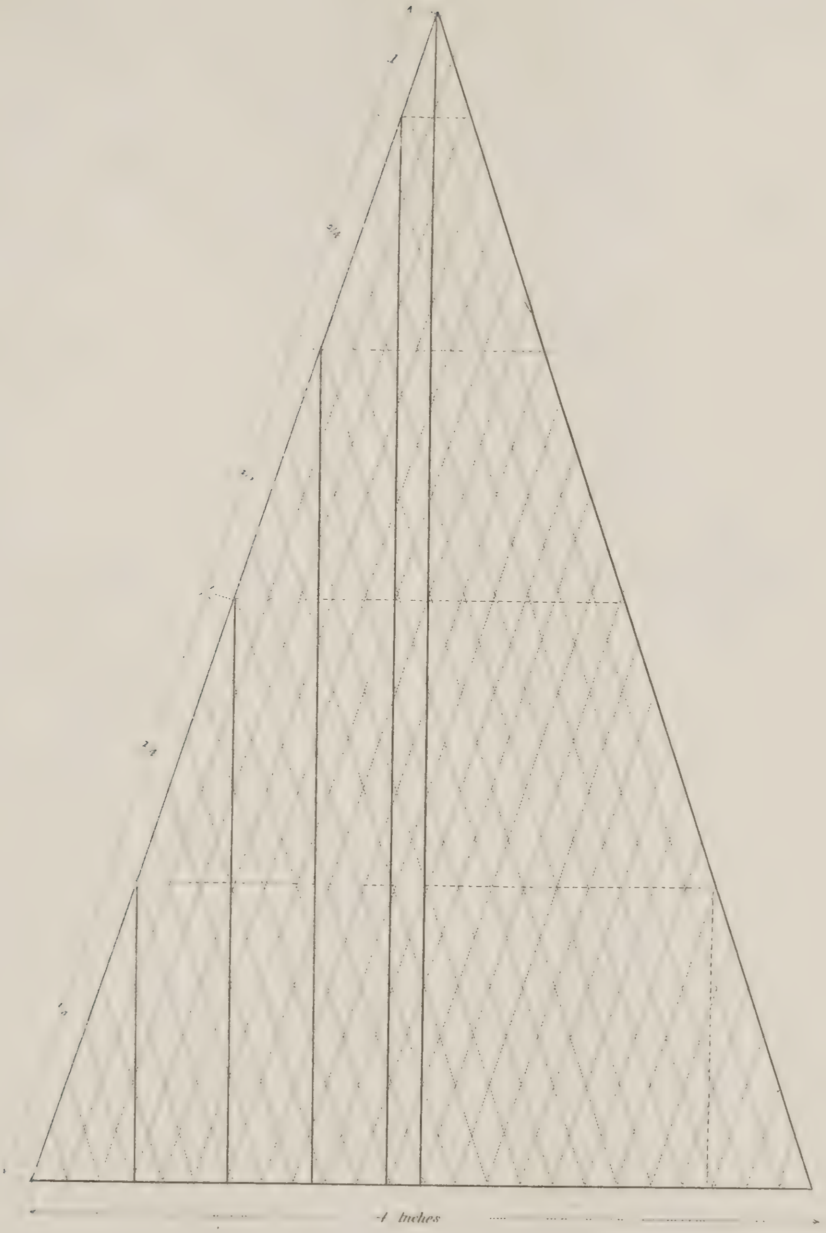
PYRAMID, No. 2.

FIG. 36.



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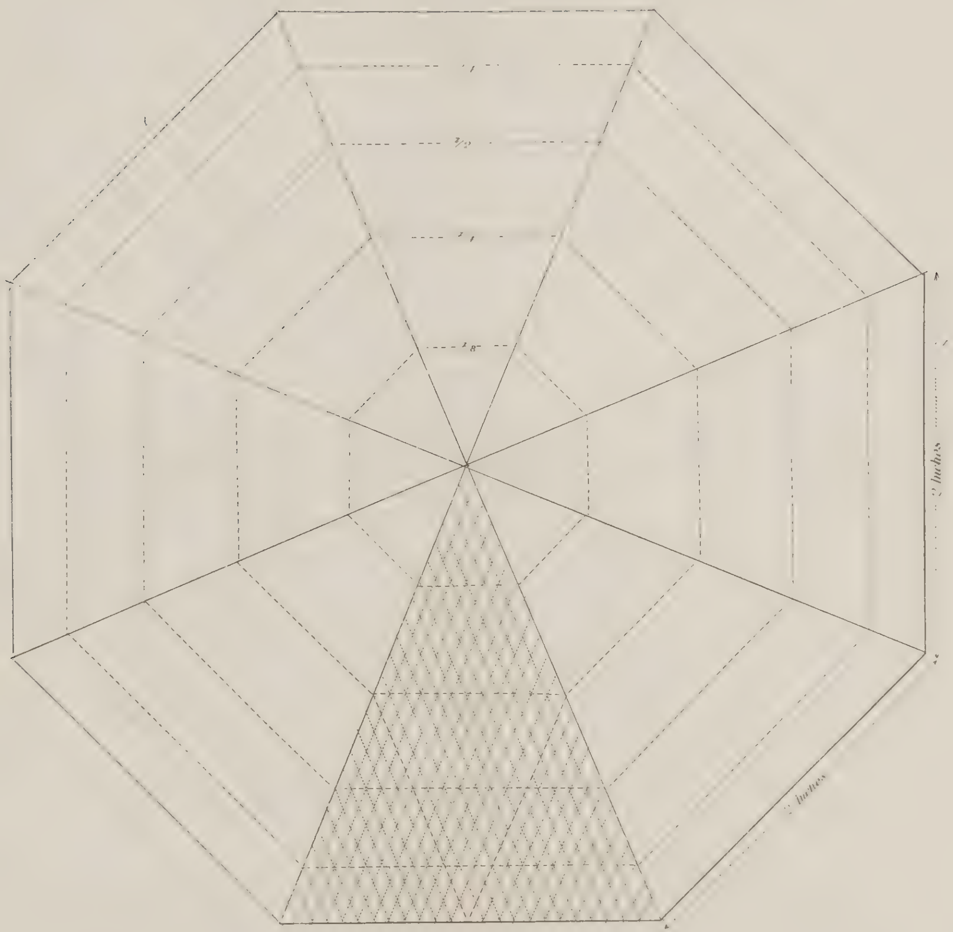
OCTAGON.

FIG. 37.



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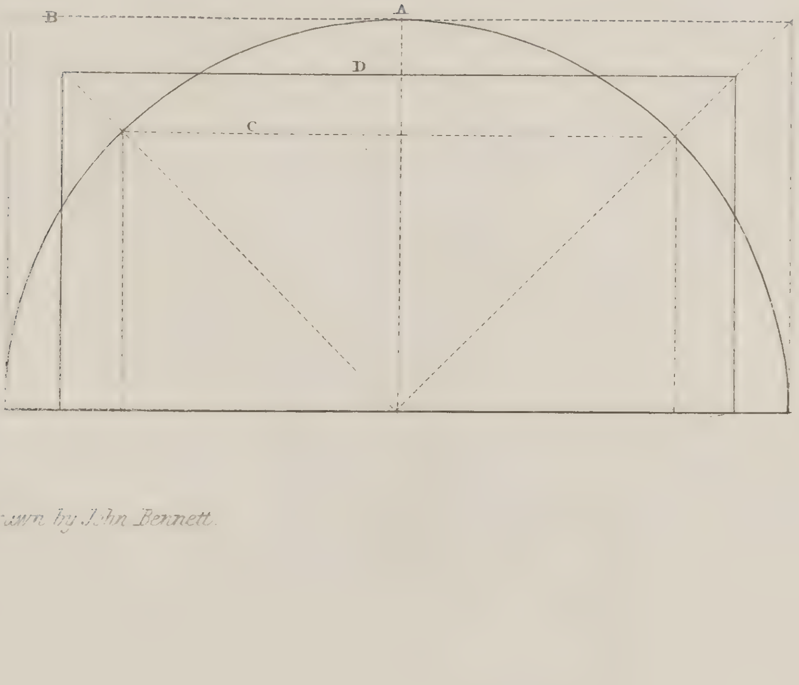
SEMI.

Q. *The Semi being given, let it be made square, and to contain the same space only*

A. A *The Semi*, B *the outer square*, C *the inner*, D *the square head required*.

FIG. 38.



ned and drawn by John Bennett.

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RHOMBUS.

A rhombus **A.B.C.D.** *being given* **E.F.G.H.** *is a Square, occupying the same space.*

FIG. 39.

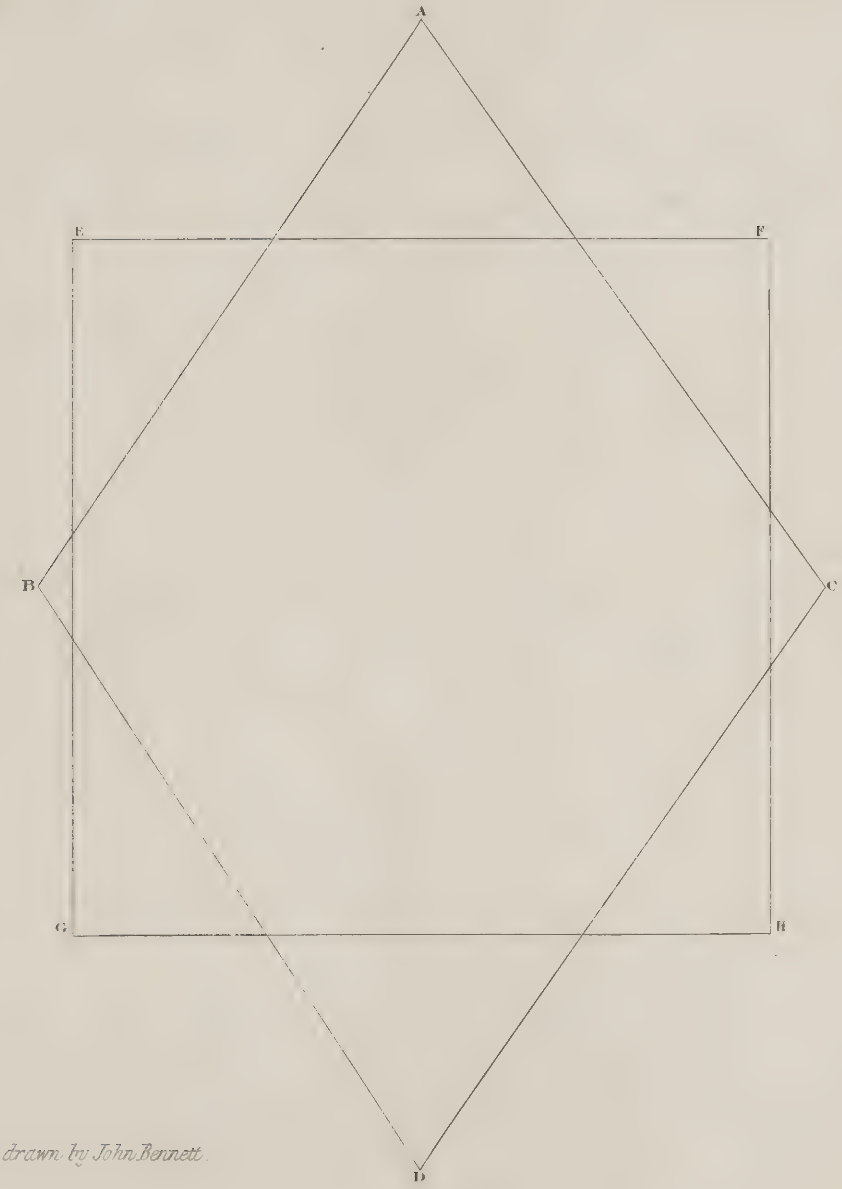

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PLATE XXXVII.

The octagon divided into aliquot parts, by the same method, which has been sufficiently before explained. The area of this octagonal figure amounts to nineteen ; consequently, each compartment must be two and three-eighths ; and the rectangle equal thereto would be two and three-eighths by one, in consequence of the base line of the octagon or the length of one of its sides being two.

PLATE XXXVIII.

The semi or half circle A being given, let it be formed with a square head as shown by the plate. Draw the dotted line B at the full extent of the semi, then the inner dotted line C, bisect AC as shown at D, and D will form the square head required.

PLATE XXXIX.

The rhombus introduced to be made into a square, occupying a similar area only. Proceed with this figure, as is explained to the fig. 19, the rhomboides : the same method which performs the operation with one, will perfectly describe the other ; the positions of the figures, as placed on the plates, nor the difference of size, will make any alteration in its solution.

PLATE XL.

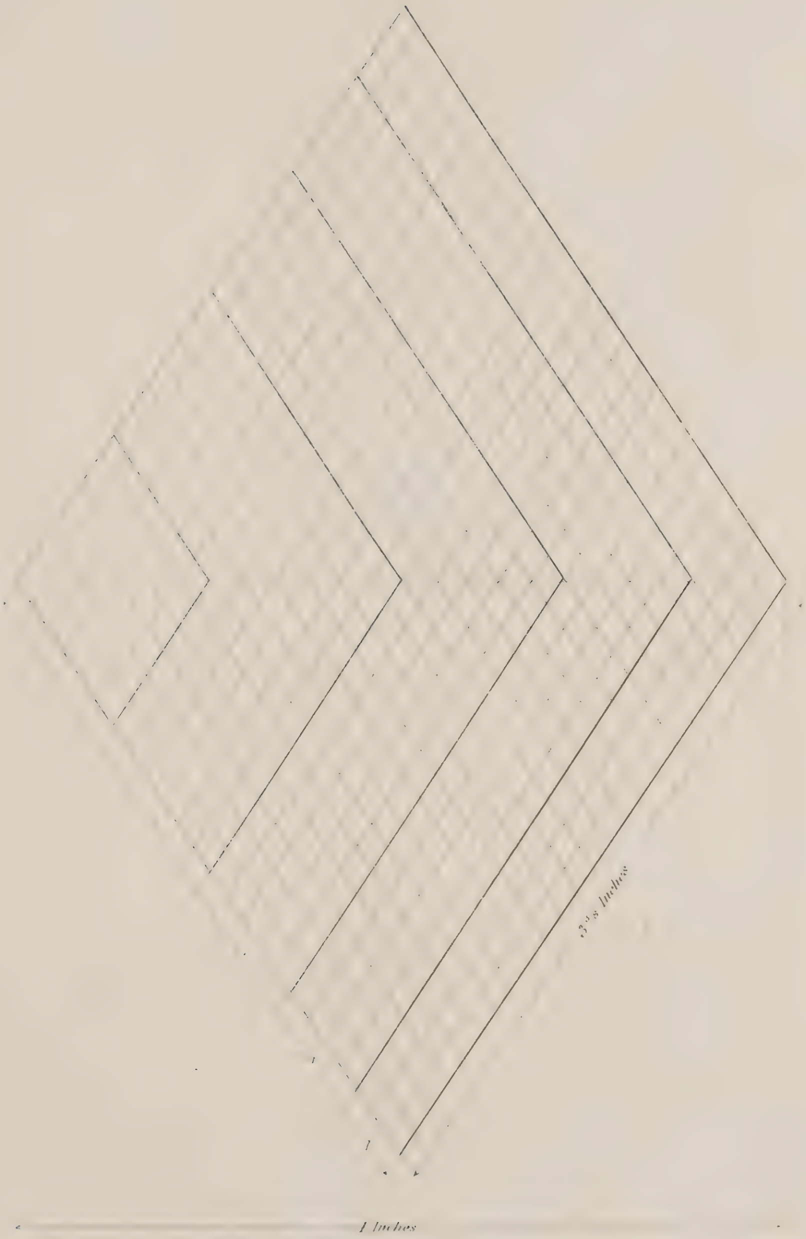
The rhombus, No. 2, divided into aliquot parts, viz., one-eighth, fourth, half, three-fourths, &c., by the methods practised with the preceding figures: the divisional parts of this are found, which are not altered in appearance from the leading figure by their diminution.

PLATE XLI.

This plate comprises three diagrams, viz., first, the cube; secondly, the surfaces, or superficies of the cube; and, thirdly, the transformation of those surfaces into another position. From these figures it must be observed, that allowing the actual measurement of another similar figure to consist of fractions, this method, in almost every instance, would serve to consolidate them, and render their solution much easier, with greater expedition and accuracy, than by taking one side only for the operation. The cube, No. 1, whose side is supposed to be twelve inches, or one foot, the superficies of which will amount to 864; the figure, No. 2, whose dimensions are $36\times24=864$; and the figure, No. 3, whose dimensions are $24\times24=576$, which is the content of the middle square: the four angles will make, viz., $12\times12=144+144=288$, which, added to 576, introduces the like solution of 864. From such variable deductions as these much valuable and interesting information might be gained from the simplicity (which these figures are chosen for) of disarranging a figure in order to obtain a true disposition of its relative quantities and component parts.

RHOMBUS, Nº 2.

FIG. 10.



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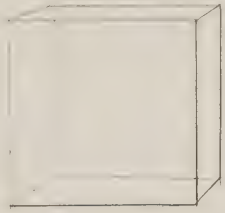
CUBE.

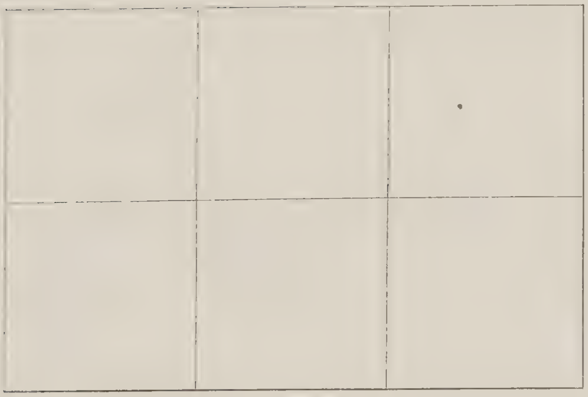
No. 1. The Cube.
2. The surfaces of the Cube.
3. The same surfaces transformed.

FIG. 41.

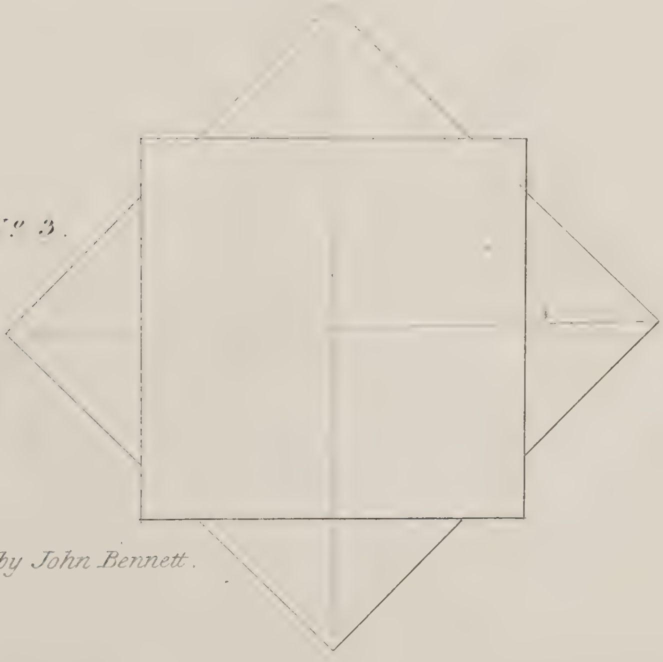
No. 1.



No. 2.



No. 3.



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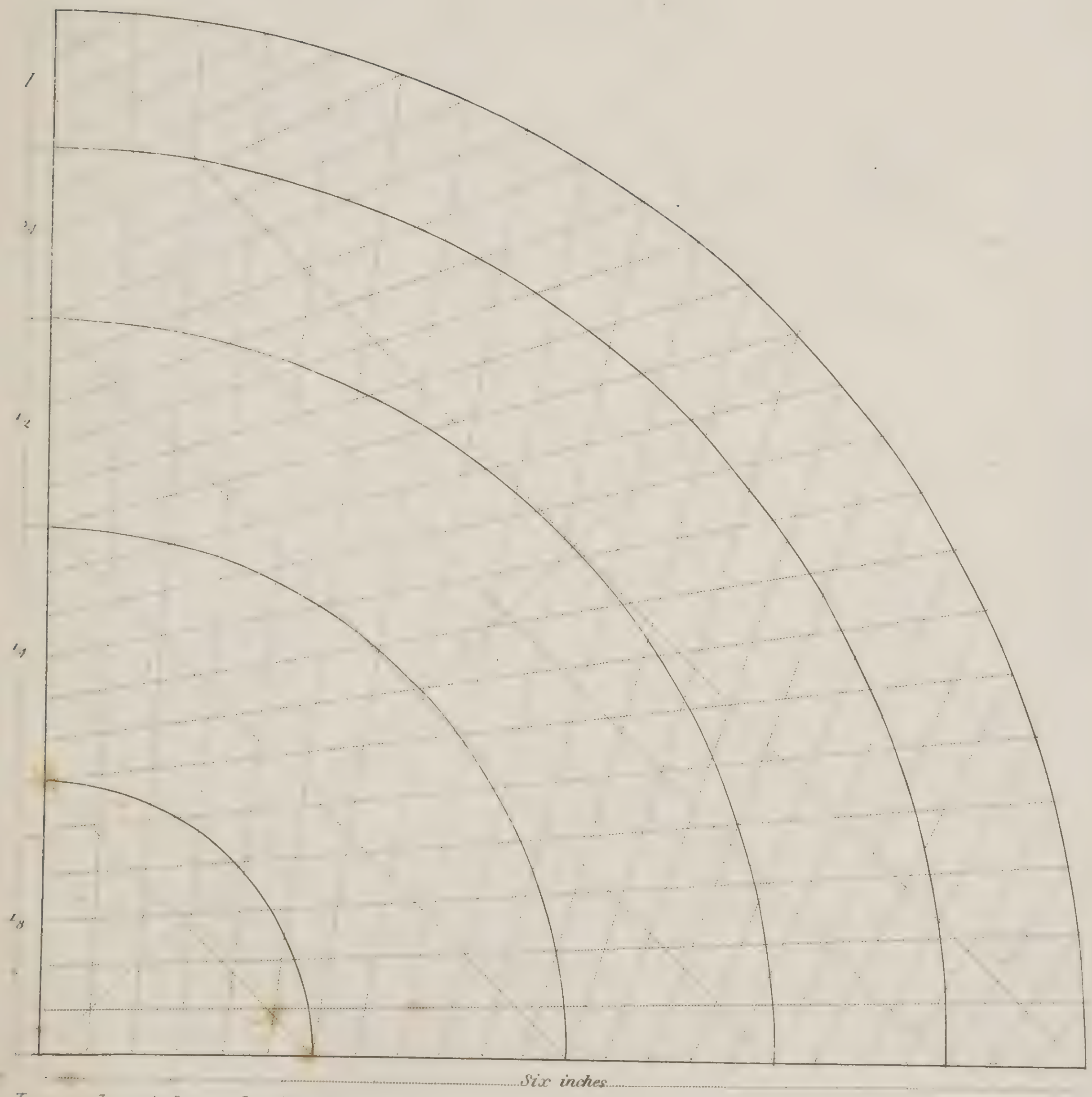
QUARTER CIRCLE,
OR
QUADRANT.

FIG. 42.



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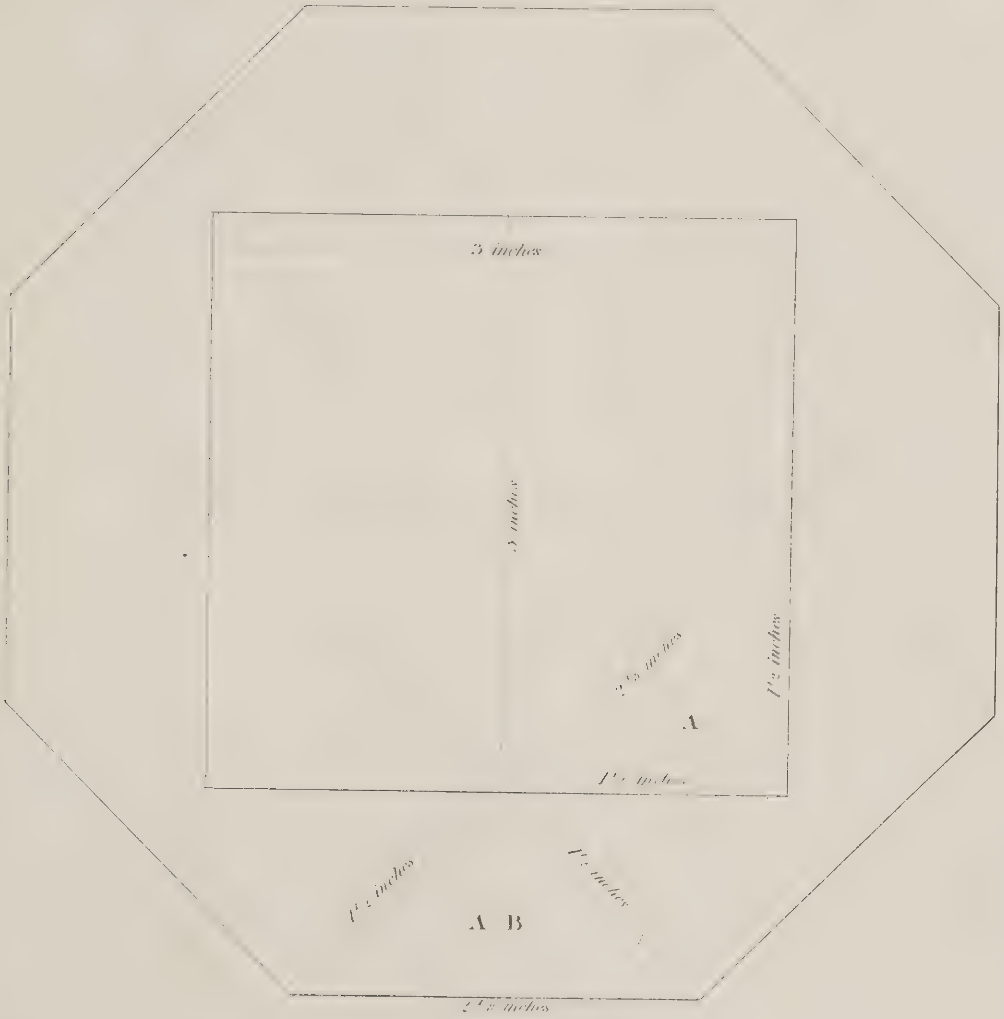
SQUARE AND OCTAGON.

The Square transformed to an Octagon, containing once and half its area; by continuing the cant A, *to* A B.

FIG. 43.



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TRIANGLES.

Qu: *An unequal sided triangle being given, required an equal sided one, containing the same space only? as* **A** *4¼, 4½, 6¾, sides.*

Ansr. **B** *_ 5 inches.*

FIG. 44.

The Triangles **A** *and* **B** *are of equal areas, although their sides are of different dimensions; which will be seen by the quantities of their sides being added together, viz.*

A	B
6¾	5
4¼	5
4½	5
15	15

The area of A = 9.5606

The area of B = 10.8253

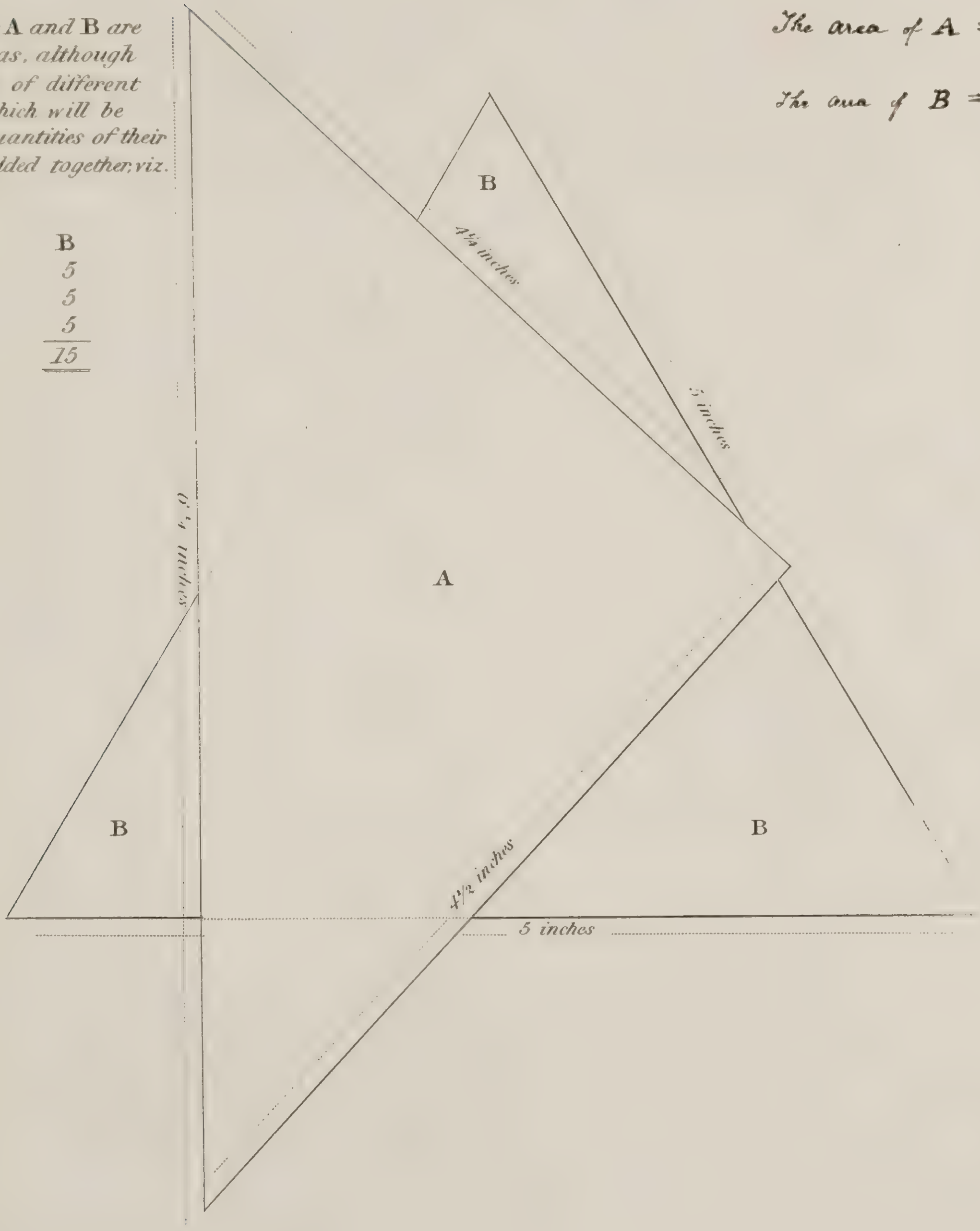

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PLATE XLII.

The quarter circle, or quadrant, divided into aliquot parts; this figure, like those preceding, is easily divided, as is shown by this method, as the others; therefore requires no further explanation to make it understood.

PLATE XLIII.

The triangle A, of the square, is equal to the triangle AB of the octagon, consequently, the four triangles must constitute half the square. This diagram may be considered of great practical utility in machinery, viz., the introduction of iron gudgeons to wood shafts for windmills, or water wheels, &c.

PLATE XLIV.

This figure comprehends the unequal-sided triangle A, whose sides are 4 and a quarter, 4 and a half, and 6 and a quarter inches; the same is in this diagram formed into the equilateral triangle B, whose area is nearly the same. The method by which this problem is performed in the margin of the plate, is as to its linear capacity; but it does not always follow with triangles that they will work to the truth with that nicety that may sometimes be required; although, in many instances, they will introduce the contents within a trifling fraction.

PLATE XLV.

The equilateral triangle, formed into two squares, of similar areas. This figure, although simple, might be so constructed as to introduce several others ; and also to show where many difficult questions arise from first appearances ; but, in this diagram, it may very readily be observed, that the upper triangle, No. 1, being reversed, and placed at the sides of A or B, will produce the quantity required for the square to either.

PLATE XLVI.

The out and inside angles. This useful problem, in taking angles of buildings, &c., may be considered of great practical utility ; for, as this diagram shows, it is merely the running out of the line of front and the side to equal distances ; and then measuring the diagonals, which must necessarily furnish the angle, let it be whatever it may : and, by the same rule, this method can be rendered of much service in the setting out of large works, where the space cannot with convenience be made applicable.

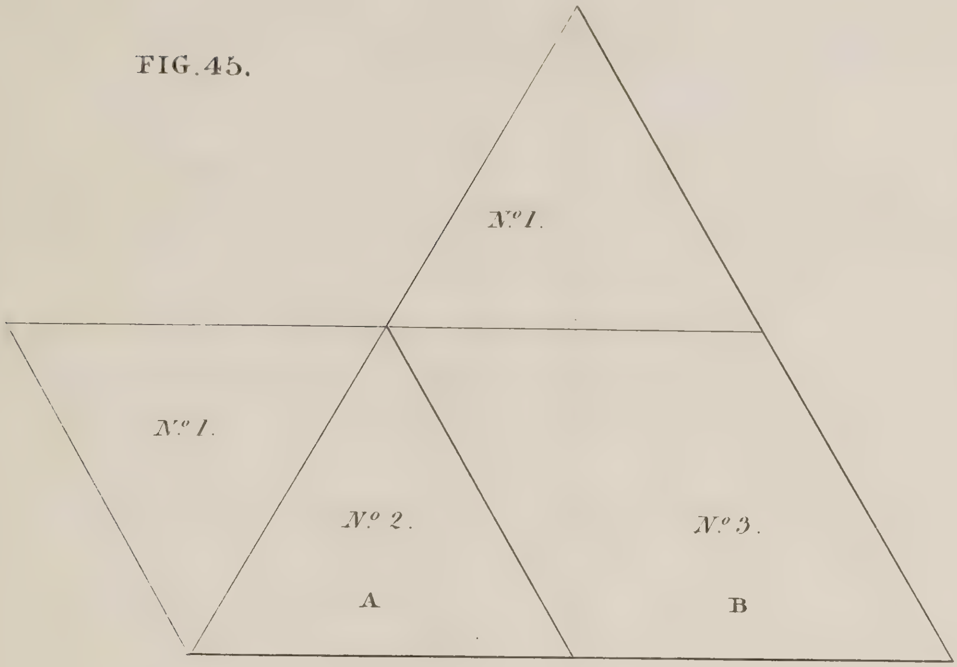
PLATE XLVII.

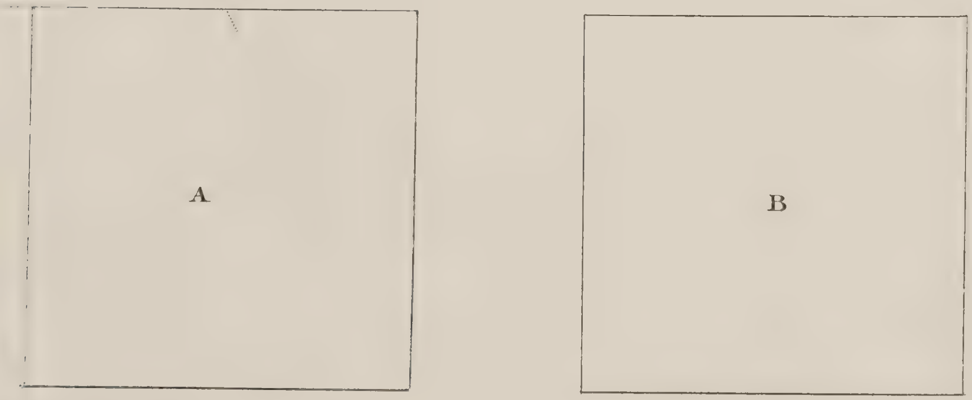
The square, No. 1, whose area is four : and No. 2 contains also the same quantity of space ; the dotted lines of which introduce the octagon. Figures No. 3 and 4 are also fabricated from them, in which are displayed much diversification of character, thereby assuming plans in the simplest

THE TRIANGLE FORMING TWO SQUARES.

From this triangle, the 2 squares **A** *and* **B**, *are constructed; containing the same space.*

FIG. 45.





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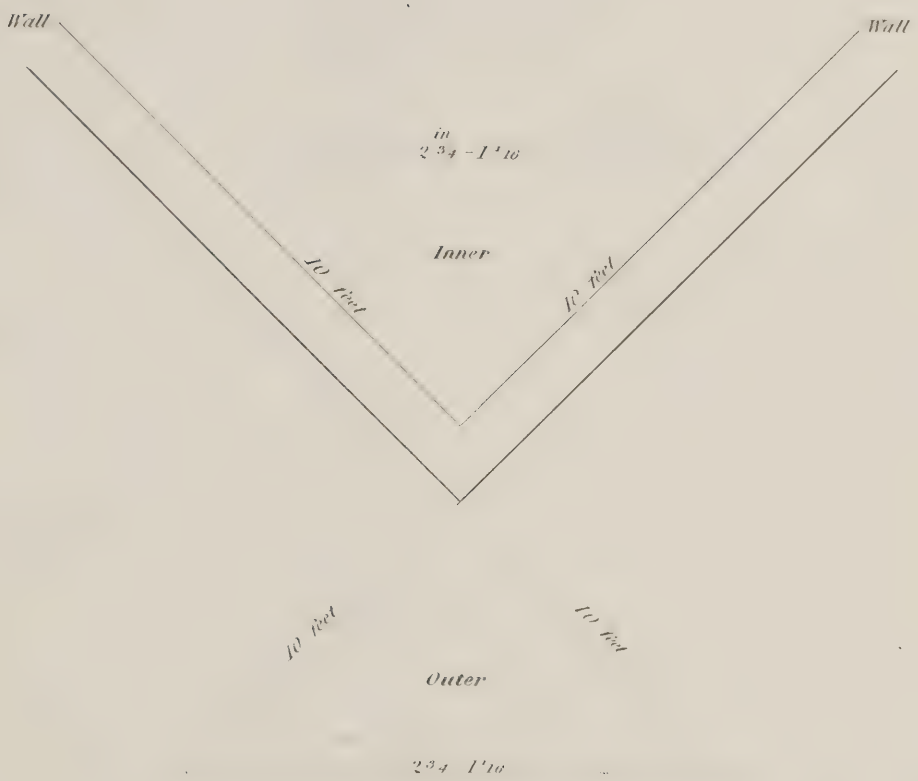
OUTER AND INNER ANGLES.

A simple method for ascertaining any Angle.

FIG. 46.



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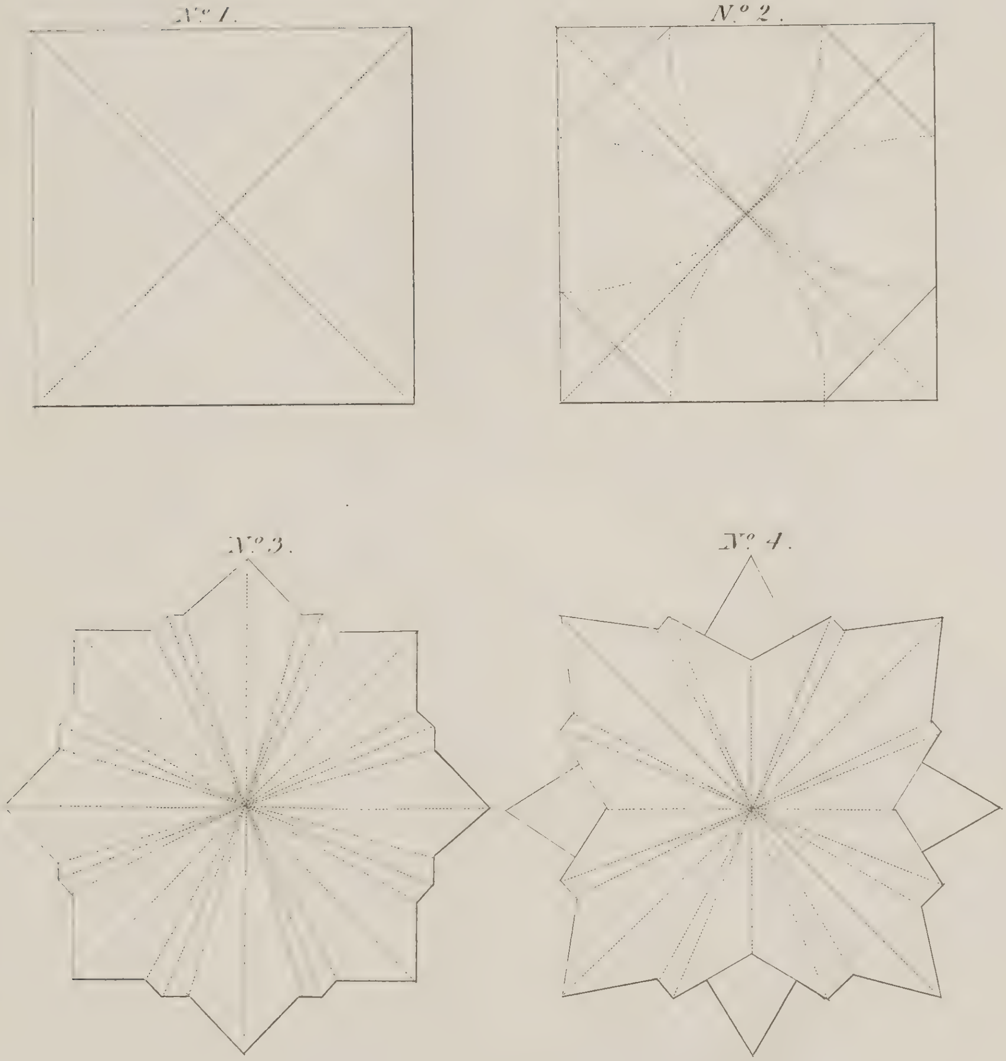
SQUARE

TRANSFORMED.

FIG. 17.



No. 1. No. 2. No. 3. No. 4.

The Compound figures Nos. 3 & 4, contain the same areas, as Nos. 1 & 2.

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COMPOUND.

This Compound fabricated from the plan A, the two squares.

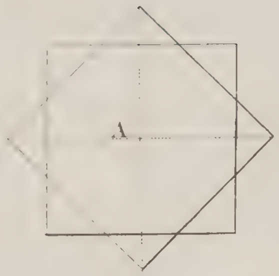
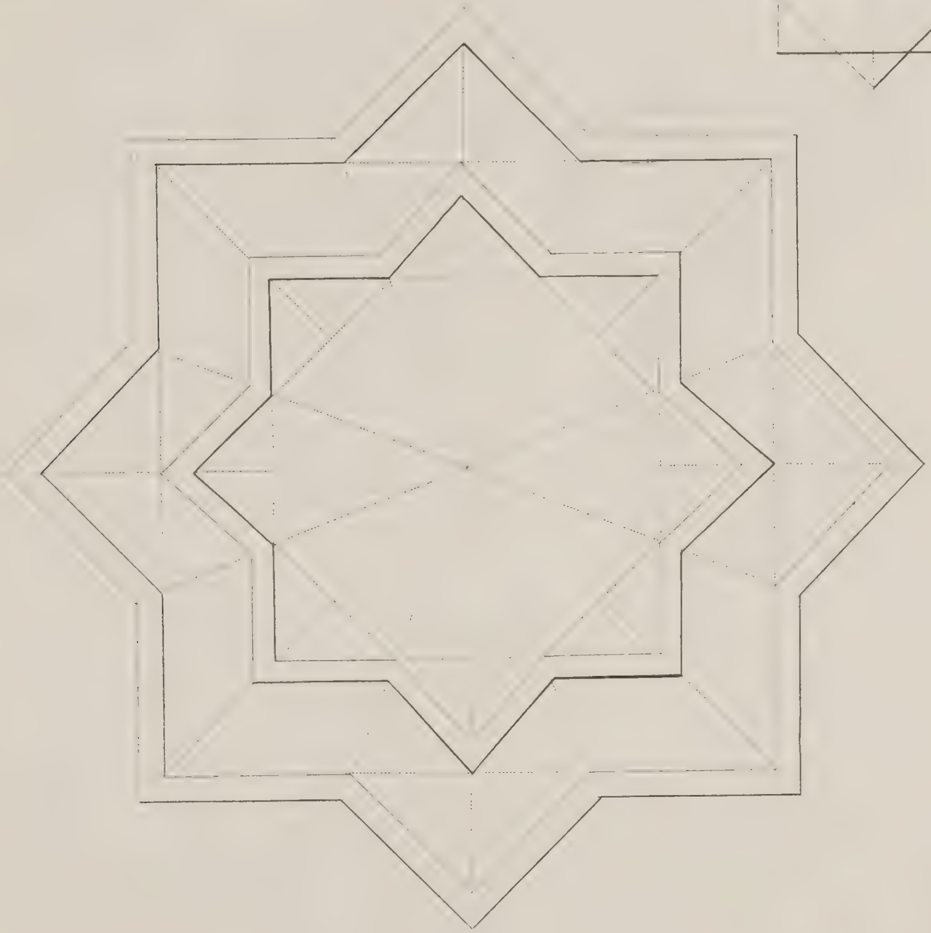


FIG. 18.



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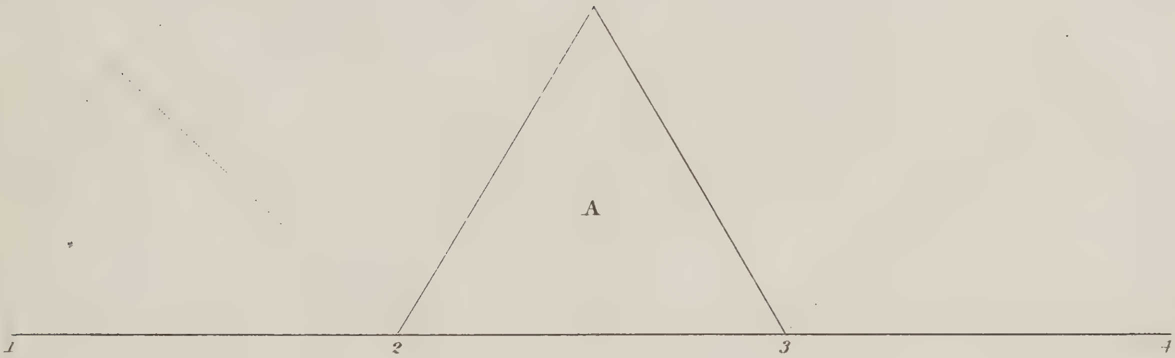
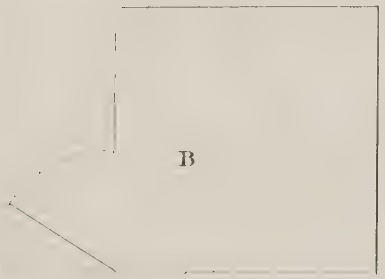
LINE.

The Line forming a triangle as **A**.

FIG. 49.



The triangle forming a Square as **B**.



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manner, suitable to some of the branches of fortification, being alike useful and ornamental. It is necessary to observe that the areas of No. 3 and No. 4 are precisely the same as No. 1 and No. 2, for such parts as are removed from the internal angles are again added to the external, leaving the original area of four for the whole of these figures.

PLATE XLVIII.

Is another plan introduced of the crossed squares, as shown by the smaller plan A. This figure may form a very useful process for the student in the science of fortification; and indeed a variety of other problems equally useful and instructive, requiring diversity of character: this plan may be altered into many more geometrical devices, and still the same area always contrived to be kept, which may serve to operate materially in some instances where any particular point requires more attention than another: the mutations upon this figure may be so constructed as to afford abundance of means requisite to fulfil any disposition required, under almost any circumstances or arrangement.

PLATE XLIX.

The straight line, formed into a triangle, as at A: this diagram, although extremely simple, would be found one of considerable difficulty to be performed in some situations upon an extensive scale; but, from a knowledge of the readiest methods, the mind, when once informed, is not readily divested of it, but much more likely to enlarge upon it by other plans of far greater intricacy. The triangle formed into a square, as at B:

Let either of the sides be divided into sixteen parts, mark off thirteen, which arrange at a right angle from the base, as shown; the other sides may then be introduced to make good the square.

PLATE L.

Is the lines A and B shown formed into squares; the line A forming one, and the line B two, which the diagrams serve sufficiently to elucidate.

PLATE LI.

The square compounded with angles. The full square A having as much abstracted from it as makes the angles CDEF; consequently the solidity is not exceeded by the alteration: but the covering surface is infinitely more. The triangle, in which the letter A is placed in this diagram, being a solid, the next on either side are vacant, as well as their opposites, the inner angles of CDBE are also vacant, and by rendering A a cubical figure, the appearance and disposition of the abstracted angles would form an interesting object, and one worthy of attention.

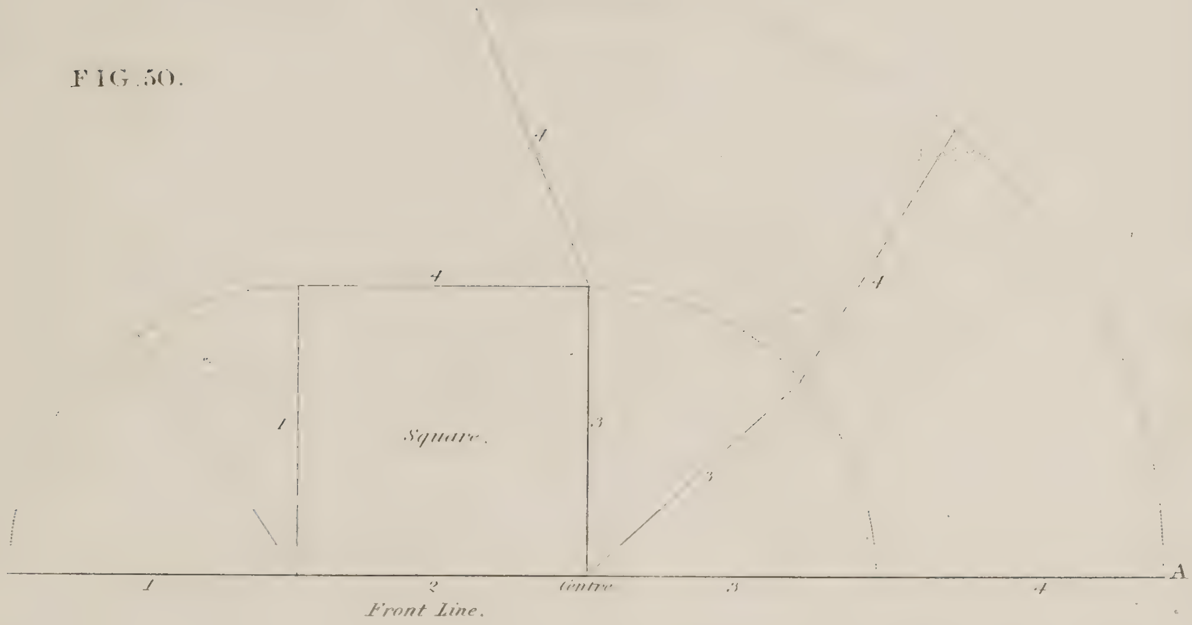
PLATE LII.

In this figure we have something similar to the foregoing, showing an octagon; and the eight triangles being reversed, serve to introduce a solid exterior, leaving the whole of the original space void: consequently the

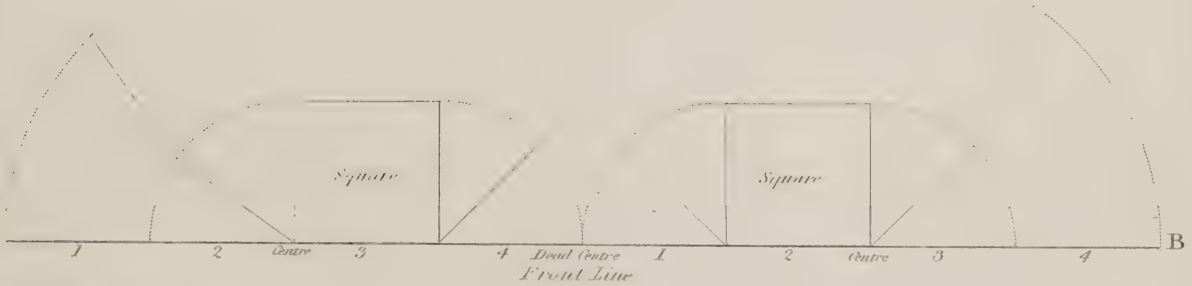
LINE.

The line A, formed into one square.

FIG. 50.



The line B, formed into two squares.



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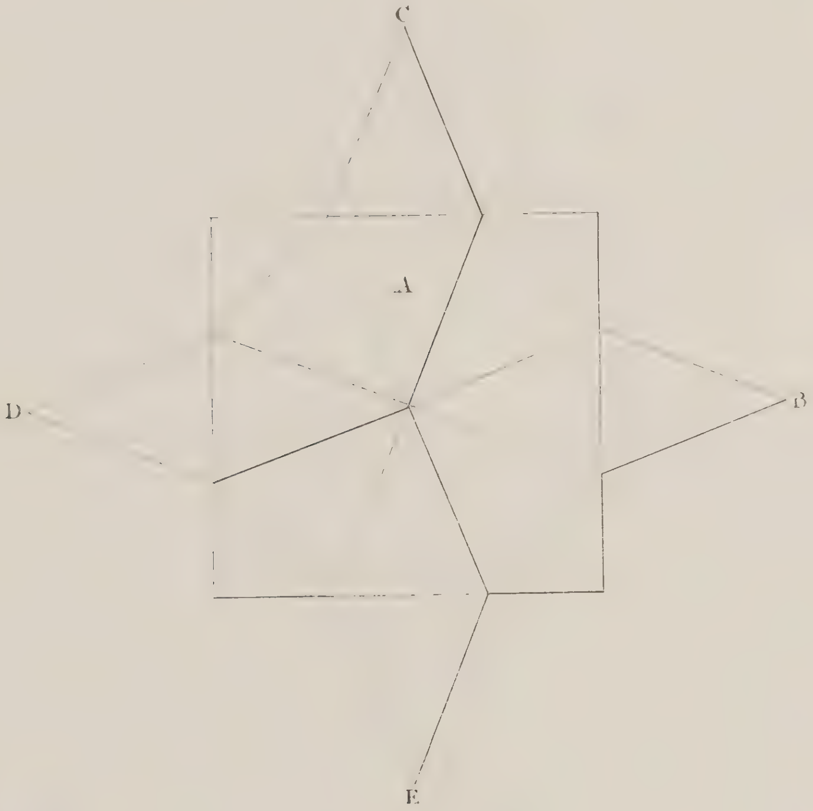
THE SQUARE COMPOUNDED WITH ANGLES.

The full square A, *having as much abstracted from it, as makes the angles* C.D.E.F; *consequently, the solidity is not exceeded by the alteration: but, the covering surface is infinitely more.*

FIG. 51.



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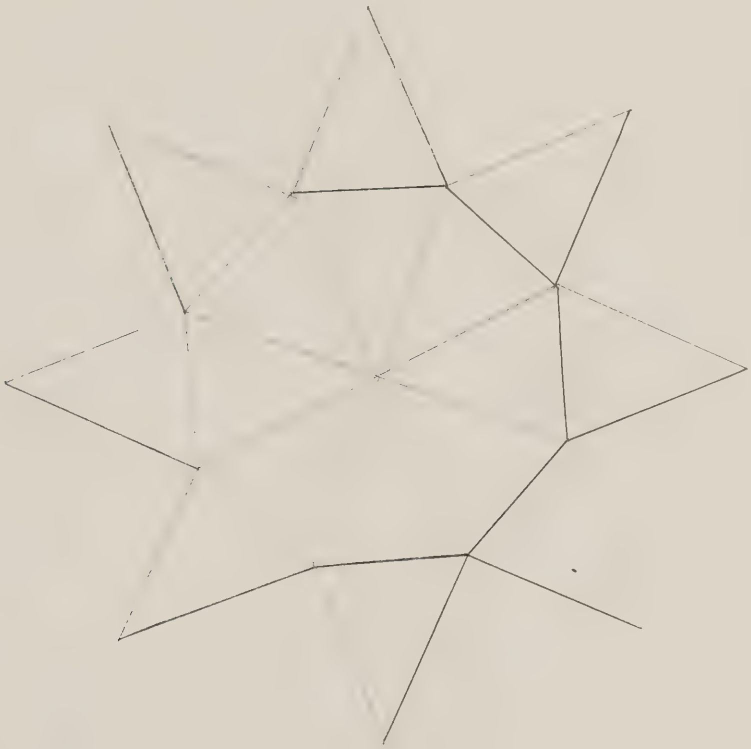
OCTAGON.

The cants, or angular divisions, of the octagonal figure reversed, and thrown outwards: as this diagram shows.

FIG. 52.



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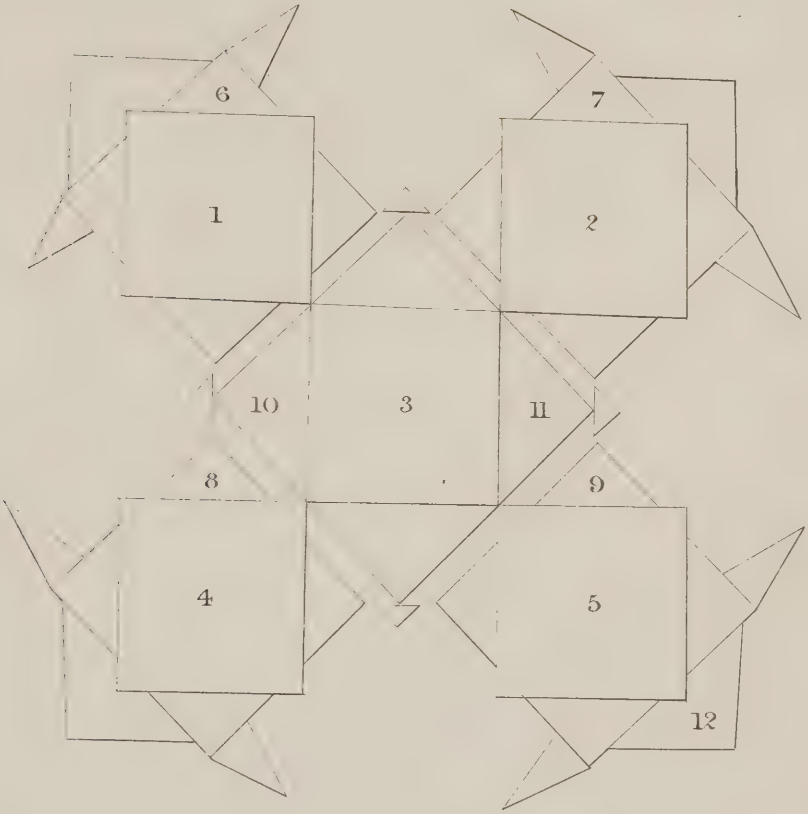
CUBE.

The surfaces of two Cubes only; transformed.

FIG. 53.



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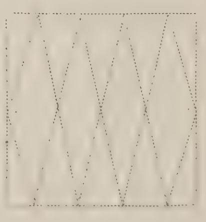

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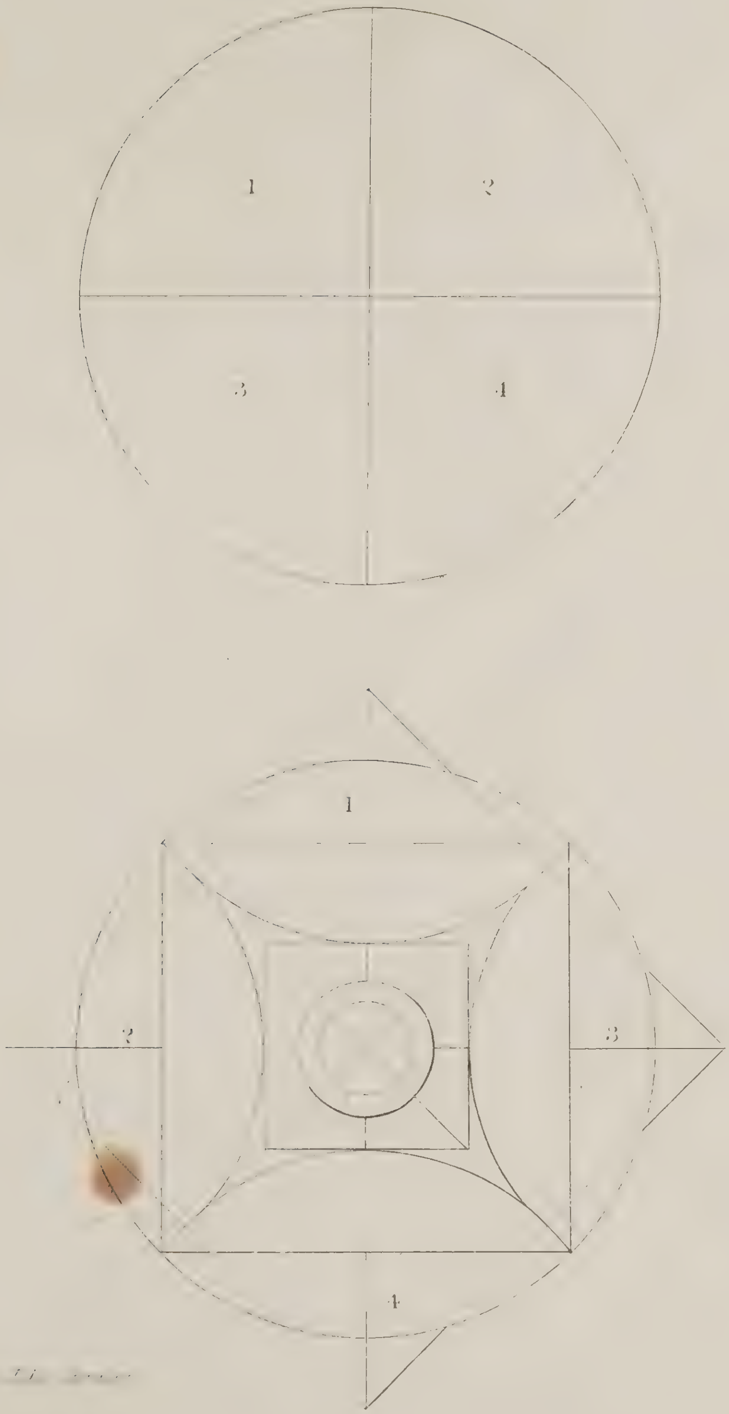
CIRCLE COMPOUNDED.

FIG. 54.



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outer figure, although it appears considerably more extensive, contains not the least particle of additional area to the inner. By introducing lines from the extremes of the points parallel to the internal diagonal lines, it will then be found to produce half a solid contained under forty-eight triangular sides, and this figure can be made to show half, or twenty-four.

PLATE LIII.

The surfaces of two cubes only, transformed. The cube has six sides, and the whole of which are uniform: therefore, the surfaces of two cubes will make altogether twelve sides; and the twelve sides are in this figure so disposed as to form the annexed plate. Numbers one, two, upper; three, centre; four and five, lower, occupy the interior. Six, seven, eight, and nine, make the angles one, two, four, five. Ten and eleven enclose the centre, and twelve make the four exterior angles. The eight protruding points are composed of half a side, as is shown by the dotted figure below. The student, in performing this figure, will be particular in observing the precise areas; and, in order to ascertain the correct admeasurement, he may readily consolidate the cubes and also the figure; he will then find their areas to correspond.

PLATE LIV.

The circle compounded, or divided into four parts; viz. one, two, three, and four; the same are disposed below in another figure, and are represented in different positions upon a similar arrangement, as Figure 53.

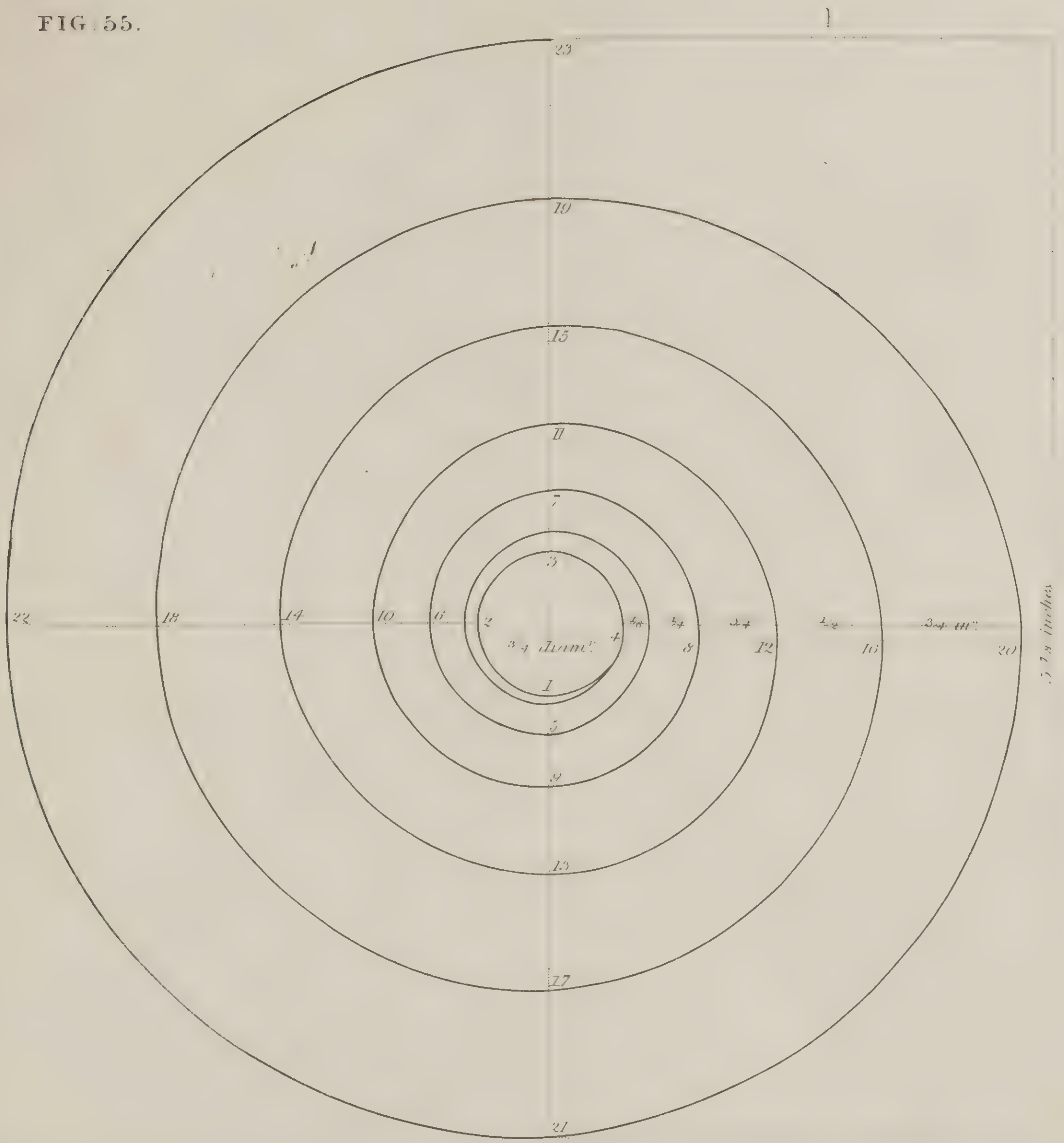
PLATE LV.

The spiral, or volute. This figure is formed in the simplest and plainest manner; and by this method any figure of like nature may thus be fabricated: it is plainly shown at the centre, or commencement of the circle, three quarters of an inch in diameter, the origin of the spiral; and the line proceeds from four to one, which sets off one division from the centre circle; another takes place at figure two, two divisions from the centre circle, three divisions at three, four divisions at four, and so upwards, until the end of the line, at twenty-three. The quarter circles, forming the spiral, may be struck with the dividers from a centre, obtained so as to take in the extreme points regularly.

SPIRAL.

One small division is extended to each quarter.

FIG. 55.



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